Cosmic Rays and the Heliospheric Plasma Environment
12 - 16 September 2011
Ruhr-Universität Bochum
Germany

Notes on drift, perpendicular, and anomalous diffusion
Part I

Adri Burger
Centre for Space Research

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Outline

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1. Ab initio approach to modulation
2. Modeling cosmic-ray transport
3. Parallel- and perpendicular diffusion coefficients
4. Turbulence power spectra
5. Drift coefficient
6. Three-dimensional drift velocity field
7. Summary and conclusions

1. Ab initio approach to modulation

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- Force-field approach to modulation requires only a single parameter to calculate an energy spectrum
- Ab initio approach needs much more…
  - Starting point is turbulence power spectrum
  - Provides input for diffusion tensor
  - Reasonable understanding of parallel- and perpendicular diffusion, but drift coefficient still a problem
  - Turbulence transport model needed to model spatial dependence of diffusion tensor
  - Etc…..

1. Ab initio approach to modulation

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  - Turbulence transport model needed to model spatial dependence of diffusion tensor
  - Etc…..
- …and then you can calculate an energy spectrum that will probably not fit data

2. Modeling cosmic-ray transport

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- Parker (1965) transport equation

$$\frac{\partial f_0(\mathbf{r},p,t)}{\partial t} = \nabla \cdot \left(\mathrm{K}^{\mathrm{S}} \cdot \nabla f_0\right) \qquad \text{diffusion}$$

$$-\mathbf{v}_d \cdot \nabla f_0 \qquad \text{drift}$$

$$-\mathbf{V}_{SW} \cdot \nabla f_0 \qquad \text{convection}$$

$$+\tfrac{1}{3}\left(\nabla \cdot \mathbf{V}_{SW}\right)\frac{p\,\partial f_0}{\partial p} \qquad \text{adiabatic cooling}$$

- Rest covered by Marius in his talk yesterday

3. Parallel- and perpendicular diffusion coefficients

- Can fit (some) data with force-field approximation but heliosphere is NOT spherically symmetric (solar wind, magnetic field, turbulence)

- Compare “radial” κ with random sweeping model of Teufel & Schlickeiser (2003, A&A) for parallel diffusion including dissipation range…

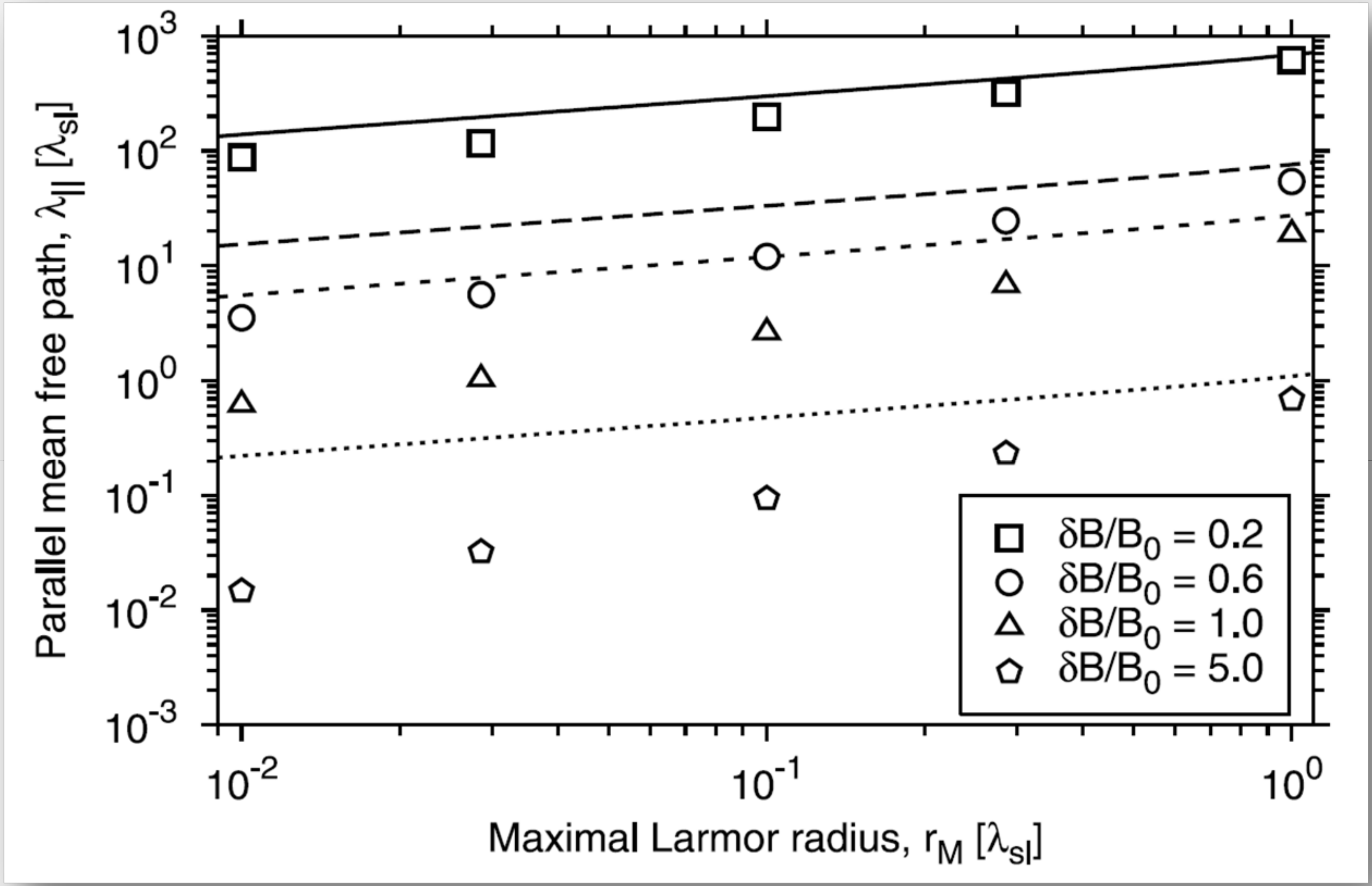
$$\kappa_{\parallel} = \left(\frac{B_0}{\delta B}\right)^2 \frac{vs}{\sqrt{\pi}(s-1)} \frac{k_{\min} R_L^2}{b} \left[\frac{b}{4\sqrt{\pi}} + \frac{2}{\sqrt{\pi}(2-s)(4-s)} \frac{b}{R^s} + \left(\frac{1}{\Gamma(p/2)} + \frac{1}{\sqrt{\pi}(p-2)} \right) \frac{b^{p-1}}{R^s Q^{p-s}} \right]$$

$$R_L = \frac{p}{B_o c}, R = k_{\min} R_L, b = \frac{v}{2\alpha v_{Alfv}} \text{ and } Q = k_D R_L$$

- …and perpendicular diffusion using approximation for nonlinear guiding center model (NLGC) of Matthaeus et al. (2003, ApJ)

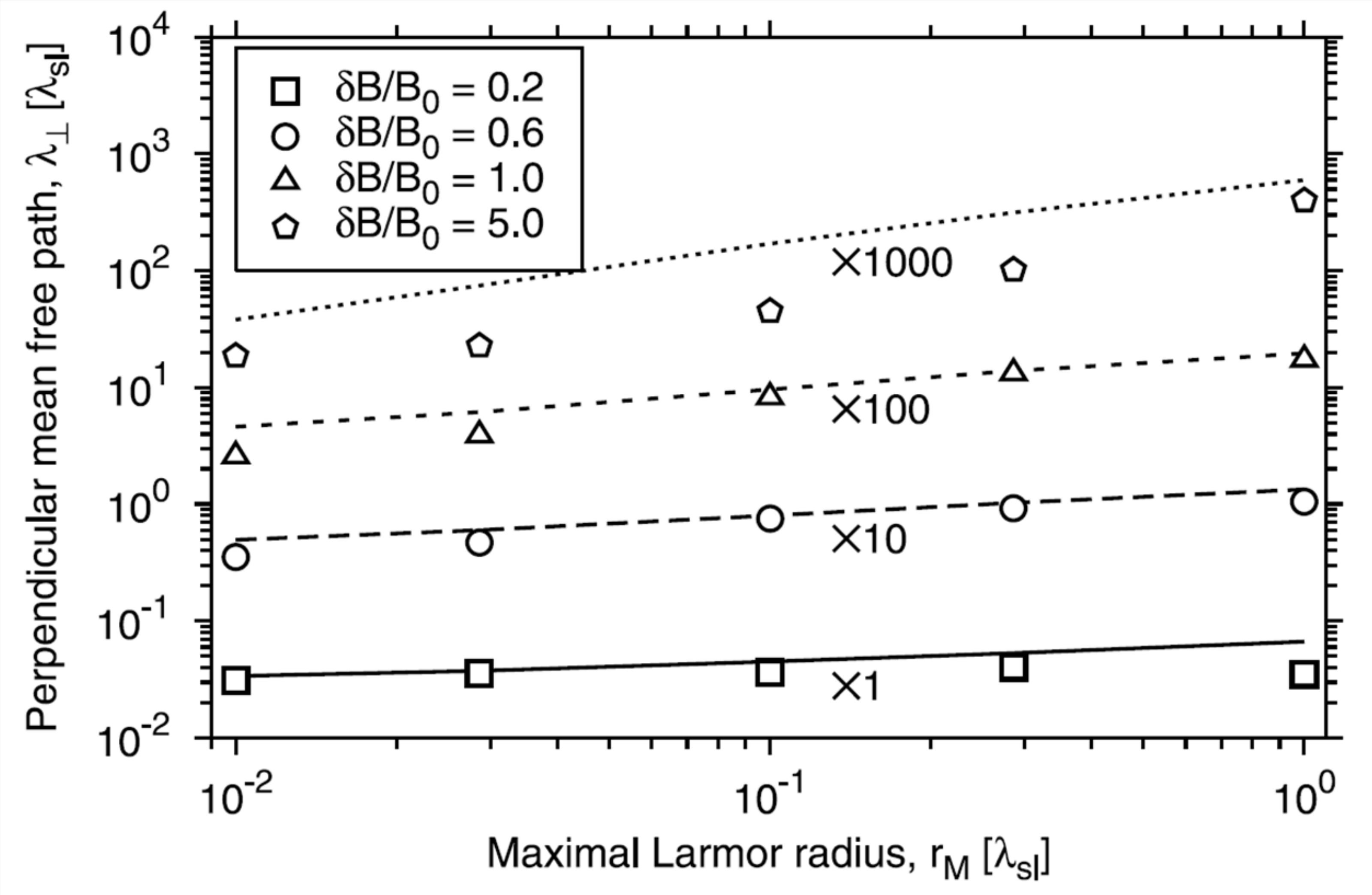
$$\kappa_{\perp} = \frac{a^2 v^2}{3B_0^2} \int \frac{S(\mathbf{k}) dk_x dk_y dk_z}{\frac{v^2}{3\kappa_{\parallel}} + k_{\perp}^2 \kappa_{\perp} + k_{\parallel}^2 \kappa_{\parallel}}$$

Comparison of QLT with numerical simulations



Minnie et al. (2007, ApJ)

Comparison of NLGC with numerical simulations



Minnie et al. (2007, ApJ)

4. Turbulence power spectra

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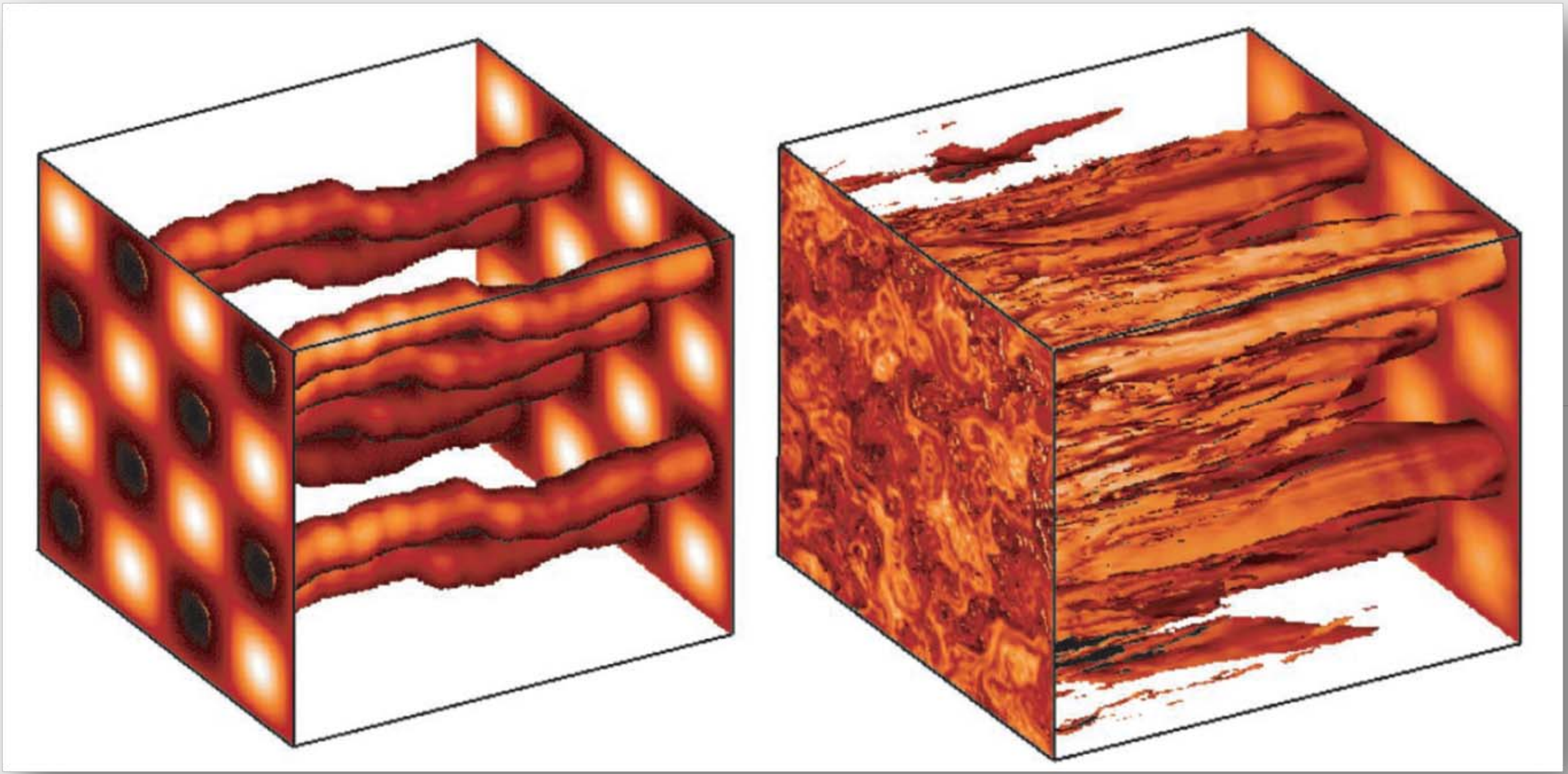
- Assume composite slab/2D turbulence



Matthaeus et al. (2003, ApJ)

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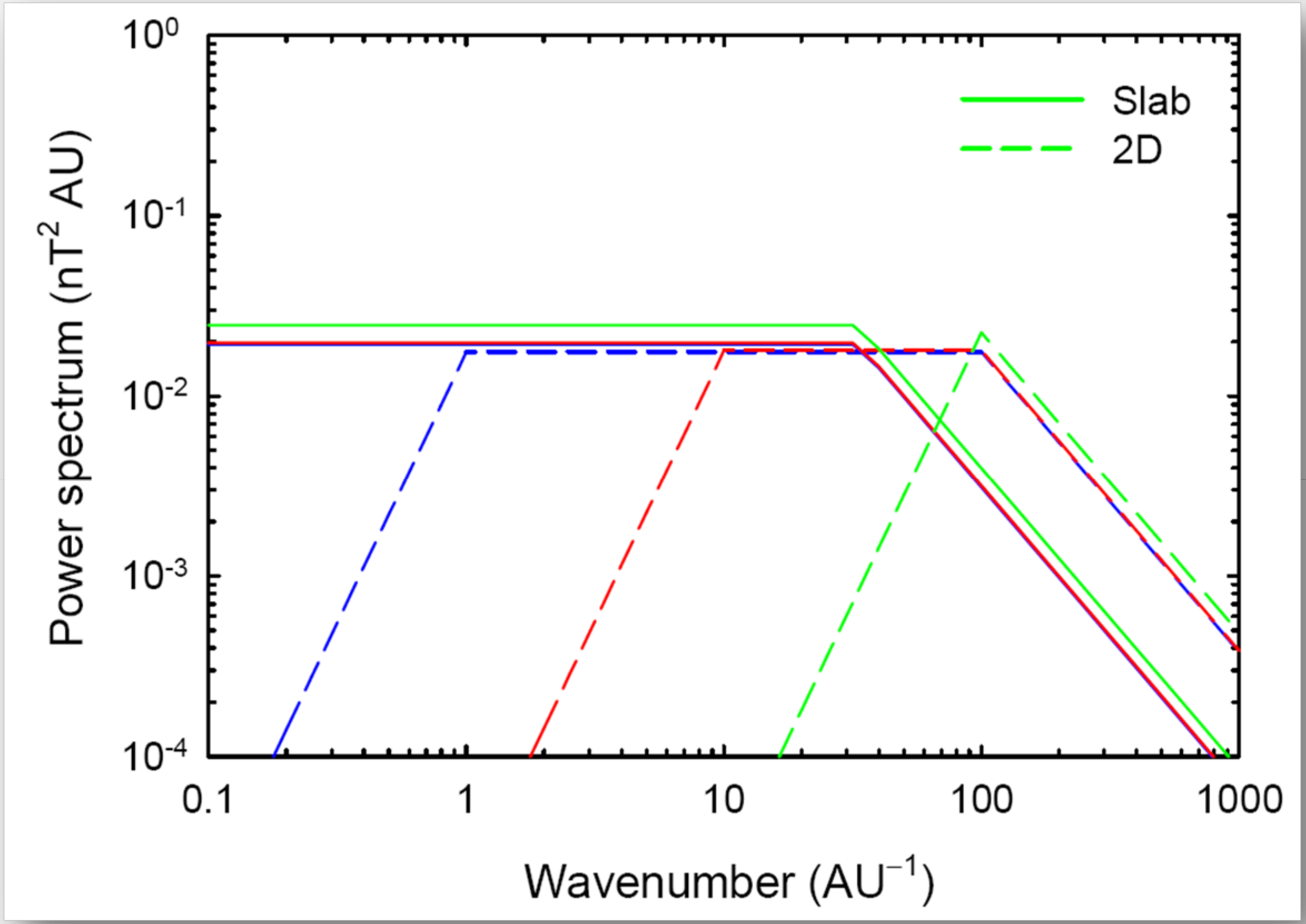
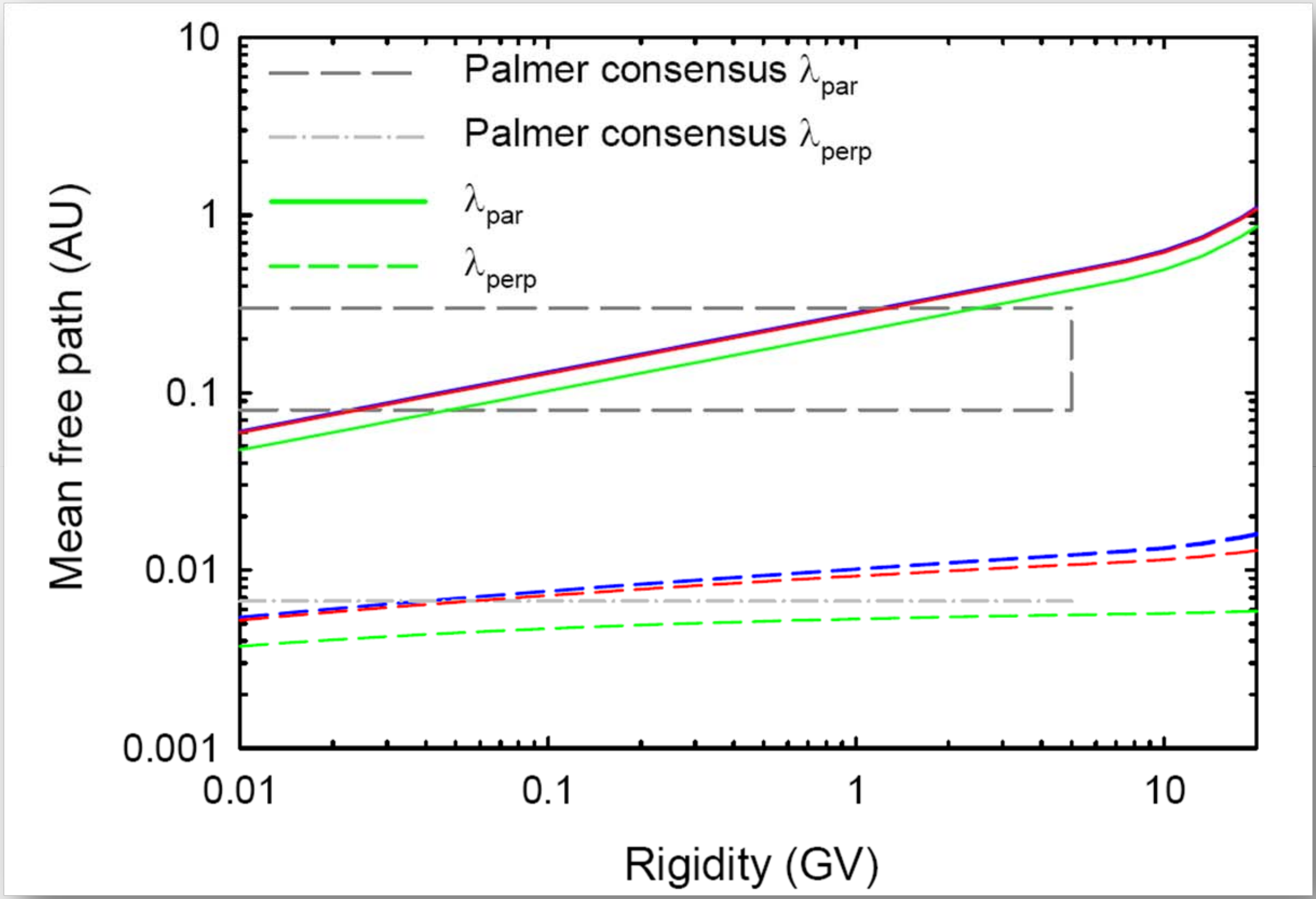
- Assume composite slab/2D turbulence (Matthaeus et al. 1990, JGR; Giacalone et al. 2006, ApJL)
- Key question is relative contribution of two components to spectrum
- Bieber et al. (1996, JGR) use Helios measurements between 0.3 and 1 AU and find dominant 2D component in **inertial range** when comparing **spectra**
- Ratio of slab to total **variance** must be calculated for given form of spectra
- Very simple form for slab spectrum (Bieber et al. 1994, ApJ) with flat energy range physically acceptable but 2D spectrum requires decreases at small wavenumbers (Matthaeus et al. 2007, ApJ)

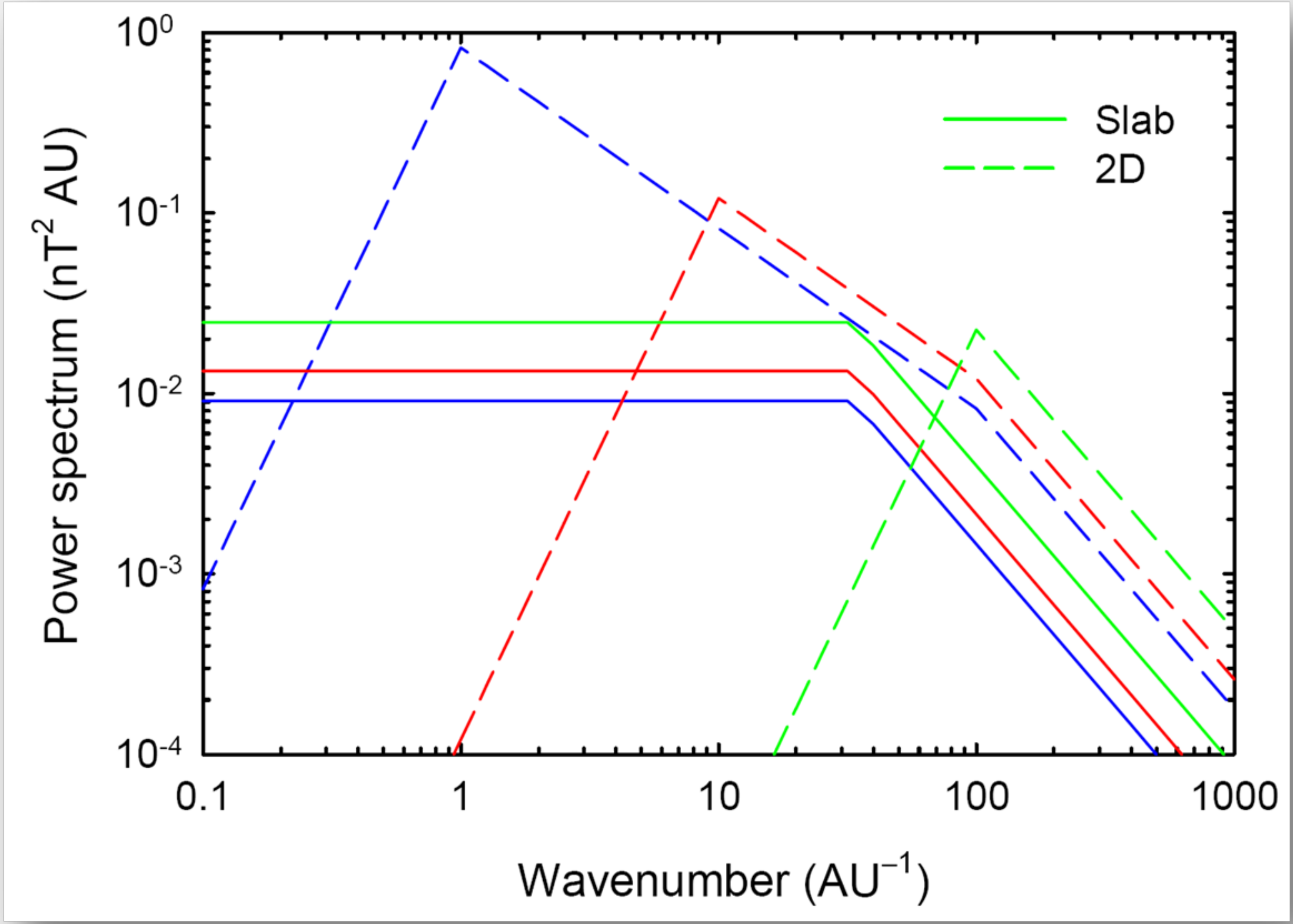
Turbulence spectra…



…and mean free paths calculated with them



Turbulence spectra…



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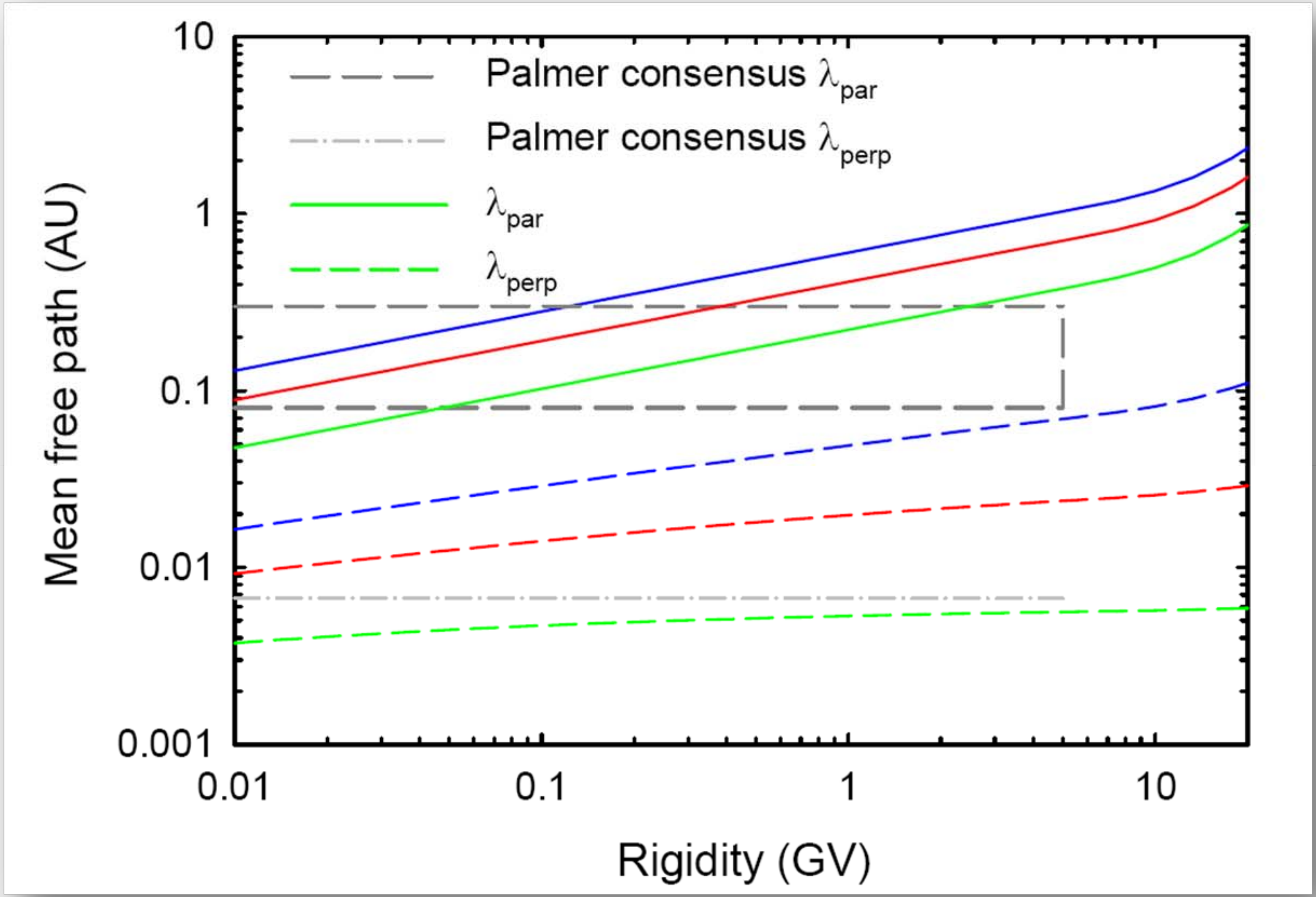
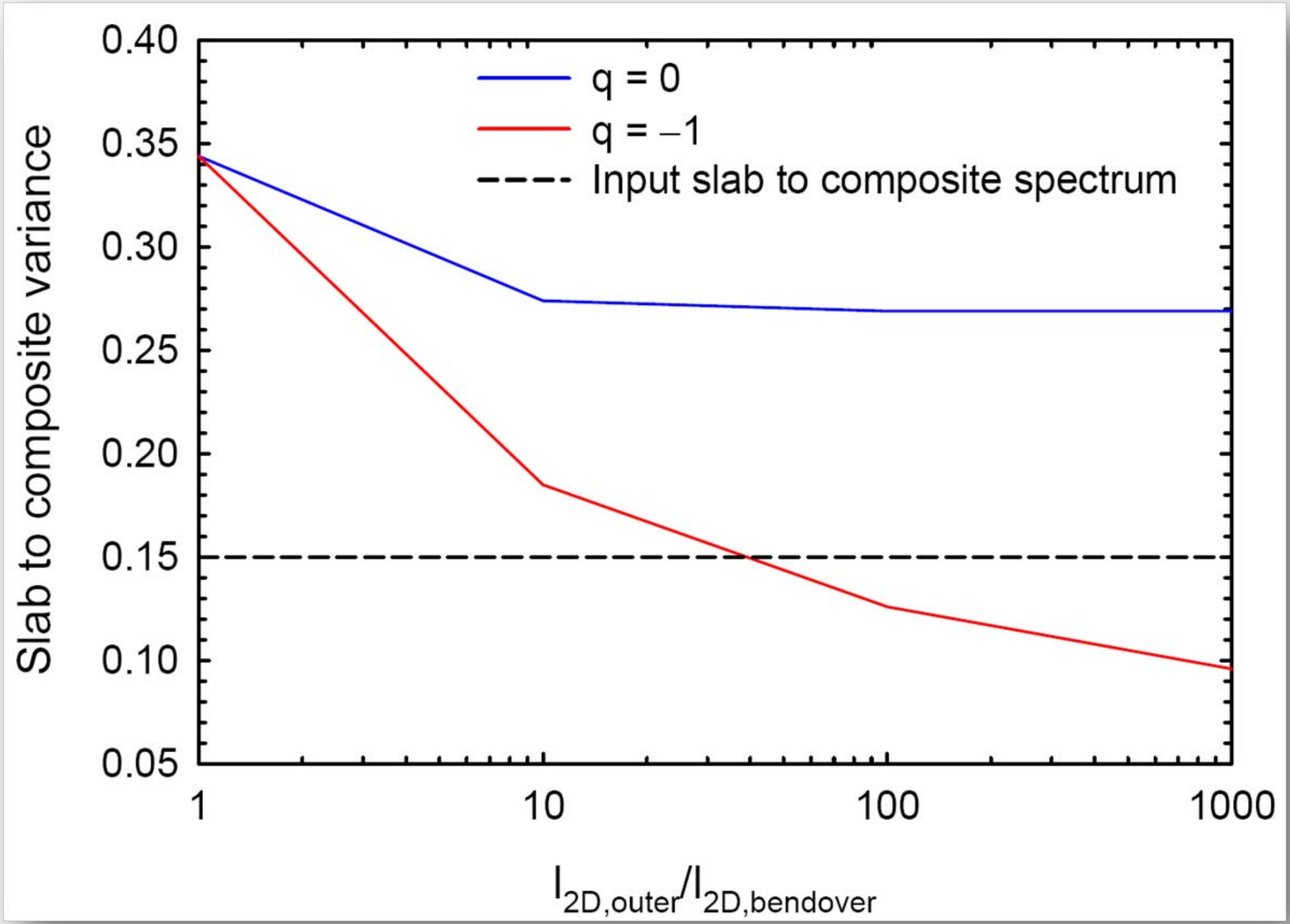
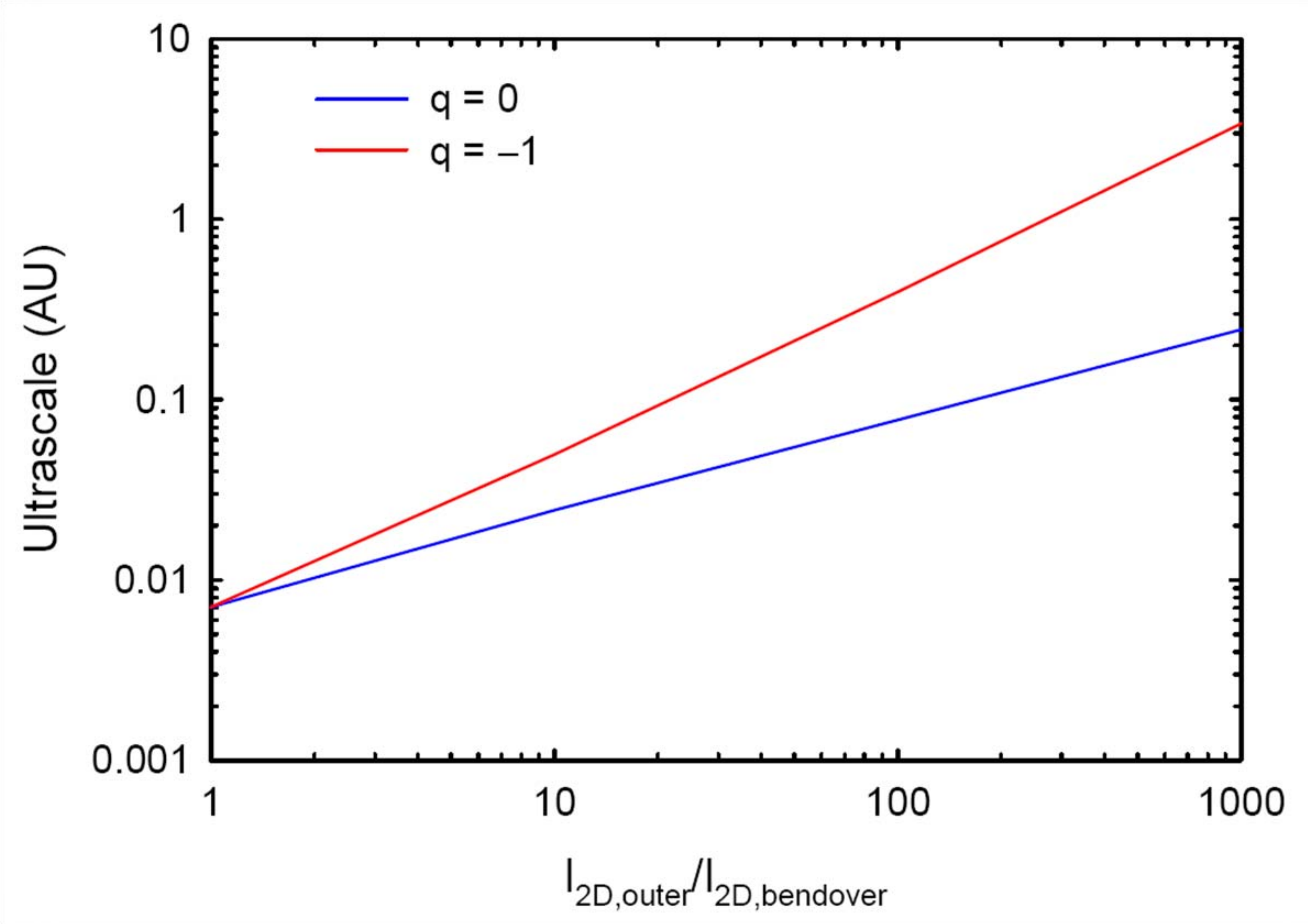


- Conceptual (or maybe a real) problem: If the spectra are kept constant at some wavenumber in the inertial range, and the ratio of slab to composite variance is prescribed, the total variance cannot be prescribed as well.

Dependence of variance ratio on inertial range spectra ratio



…but that's not the end of the story



5. Drift coefficient

- No successful theory yet for drift coefficient (Burger & Visser 2010, ApJ; Shalchi 2011, PhysRevE)
- "Standard" form for drift coefficient (e.g. Forman et al. 1974, ApJ; Jokipii 1993, ICRC):

$$\kappa_A^{ws} = \frac{pv}{3qB} = \frac{vR_L}{3} = \frac{\beta P}{3B},$$

$$\kappa_A = \frac{\beta P}{3B} \frac{(\omega\tau)^2}{1+(\omega\tau)^2}$$

- What should we use for $\omega\tau$?

- Scattering parameter (Bieber & Matthaeus, 1997, ApJ) (BAM) and FLRW diffusion coefficient (Matthaeus et al. 1995, PRL; 2007, ApJ)

$$\omega\tau = \frac{2}{3}\frac{R_L}{D_\perp}, \quad D_\perp = \frac{1}{2}\left[D_{sl} + \sqrt{D_{sl}^2 + 4D_{2D}^2}\right]$$

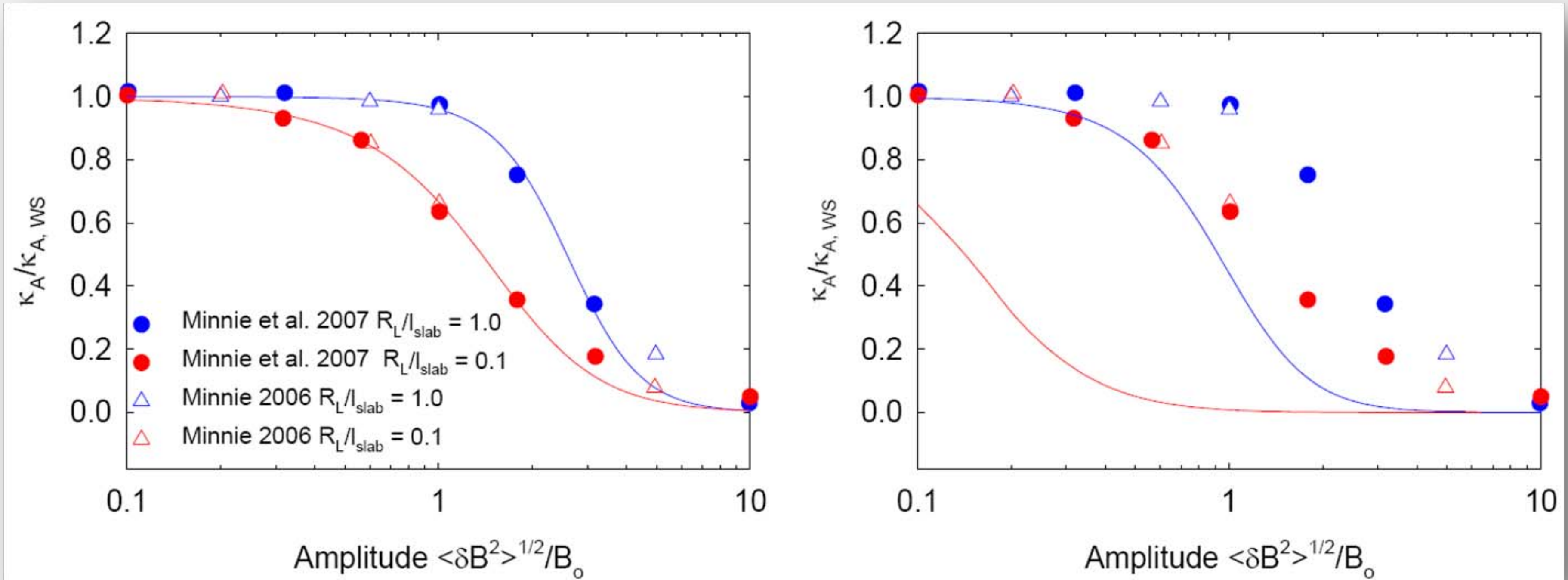
$$D_{sl} = \frac{1}{2}\frac{\delta B_{sl}^2}{B_0^2} l_{sl}, \quad D_{2D} = \sqrt{\frac{\delta B_{2D}^2}{B_0^2}}\, l_{ultra}$$

$$l_{ultra} = \sqrt{\frac{\int d^2k S_{2D}(k)/k^2}{\int d^2k S_{2D}(k)}}$$

Functional approach to drift reduction for slab/2D structure

- Modified scattering parameter (Visser 2010, MSc; Burger & Visser 2010, ApJ)

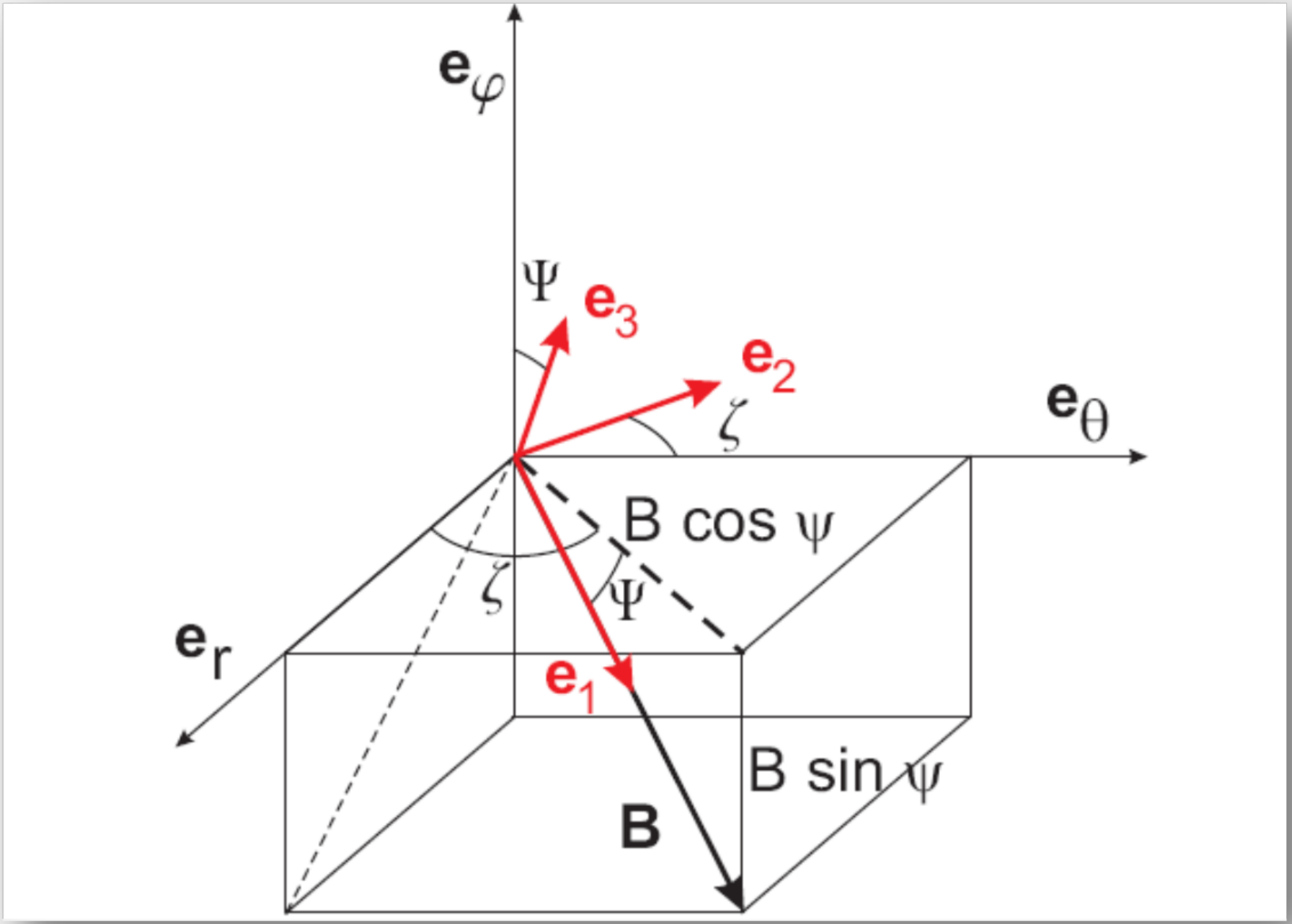
$$\omega\tau = \frac{8}{3}\frac{\left(R_L / l_{sl}\right)^{0.45}}{\left(D_\perp / l_{sl}\right)^{g}}, \qquad g = 0.3\log\left(R_L / l_{sl}\right) + 0.8$$



- Functional approach (left) vs. BAM (right)

Drift coefficient and diffusion tensor

- Field-aligned and spherical coordinates



- Diffusion tensor in field-aligned…

$$\mathrm{K}' = \begin{bmatrix} \kappa_{||} & 0 & 0 \\ 0 & \kappa_{\perp,2} & \kappa_A \\ 0 & -\kappa_A & \kappa_{\perp,3} \end{bmatrix} = \begin{bmatrix} \kappa_{||} & 0 & 0 \\ 0 & \kappa_{\perp,2} & 0 \\ 0 & 0 & \kappa_{\perp,3} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \kappa_A \\ 0 & -\kappa_A & 0 \end{bmatrix}$$

- …and in spherical coordinates

$$\mathrm{K} = \begin{bmatrix} \kappa_{rr} & \kappa_{r\theta} & \kappa_{r\phi} \\ \kappa_{\theta r} & \kappa_{\theta\theta} & \kappa_{\theta\phi} \\ \kappa_{\phi r} & \kappa_{\phi\theta} & \kappa_{\phi\phi} \end{bmatrix} = \mathrm{K}^s + \mathrm{K}^a$$

- Anti-symmetric part of diffusion tensor…

$$\mathrm{K}^{a} = \begin{bmatrix} 0 & -\kappa_A \sin\Psi & -\kappa_A \cos\Psi \sin\zeta \\ \kappa_A \sin\Psi & 0 & \kappa_A \cos\Psi \cos\zeta \\ \kappa_A \cos\Psi \sin\zeta & -\kappa_A \cos\Psi \cos\zeta & 0 \end{bmatrix}$$

- …recast as drift velocity in transport equation (Minnie et al. 2007, ApJ; Aris, 1989; Vectors, Tensors, and the Basic Equations of Fluid Mechanics)

$$-\nabla \cdot \mathrm{K}^{a} = \nabla \times \left[\kappa_A \left(\cos\Psi \cos\zeta \, \mathbf{e}_r + \cos\Psi \sin\zeta \, \mathbf{e}_\theta - \sin\Psi \, \mathbf{e}_\phi \right) \right]$$

$$\equiv \nabla \times \left(\kappa_A \mathbf{e}_B \right)$$

$$\equiv \mathbf{v}_D$$

Key questions about the drift coefficient

(Also discussed recently by Shalchi 2011, PhysRevE)

- Can one calculate a drift coefficient for the case of a uniform background magnetic field?
  - Yes: See Giacalone et al. (1999, 26th ICRC)
- What form should be used to calculate it, $<\Delta x \Delta y>/(2t)$ or $<\Delta x\, v_y>$?
  - $<\Delta x\, v_y> = - <\Delta y\, v_x>$: See Giacalone et al. (1999, 26th ICRC)

- Consider particles in a uniform magnetic field $\boldsymbol{B} = B_0\mathbf{e}_z$ (Burger & Visser 2010, ApJ)
- Components of velocity…

$$v_x(t) = v_\perp \cos(\omega_c t + \delta)$$

$$v_y(t) = -v_\perp \sin(\omega_c t + \delta)$$

- …and displacement

$$\Delta x(t) = (v_\perp / \omega_c)\left[\sin(\omega_c t + \delta) - \sin\delta\right]$$

$$\Delta y(t) = (v_\perp / \omega_c)\left[\cos(\omega_c t + \delta) - \cos\delta\right]$$

- Cyclotron frequency and perpendicular velocity component in terms of pitch angle μ

$$\omega_c = qB/m \quad \text{and} \quad v_\perp = v\sqrt{1-\mu^2}$$

- Calculate ensemble averages: “standard” form yields

$$\langle \Delta x \Delta y \rangle = \langle \Delta y \Delta x \rangle = 0$$

- Approach of Giacalone et al. yields

$$\langle v_x \Delta y \rangle = -\langle v_y \Delta x \rangle = (vR_L/3)(1-\cos\omega_c t)$$

- Calculate two-point (time) correlation function…

$$\left\langle v_x(t)v_y(0)\right\rangle = -\left\langle v_y(t)v_x(0)\right\rangle = \left(v^2/3\right)\sin\omega_c t$$

- …and introduce de-correlation function to find

$$\int_0^\infty \left\langle v_x(t)v_y(0)\right\rangle e^{-\nu t}dt \overset{\nu\to 0}{=} \frac{vR_L}{3}$$

6. Three-dimensional drift velocity field

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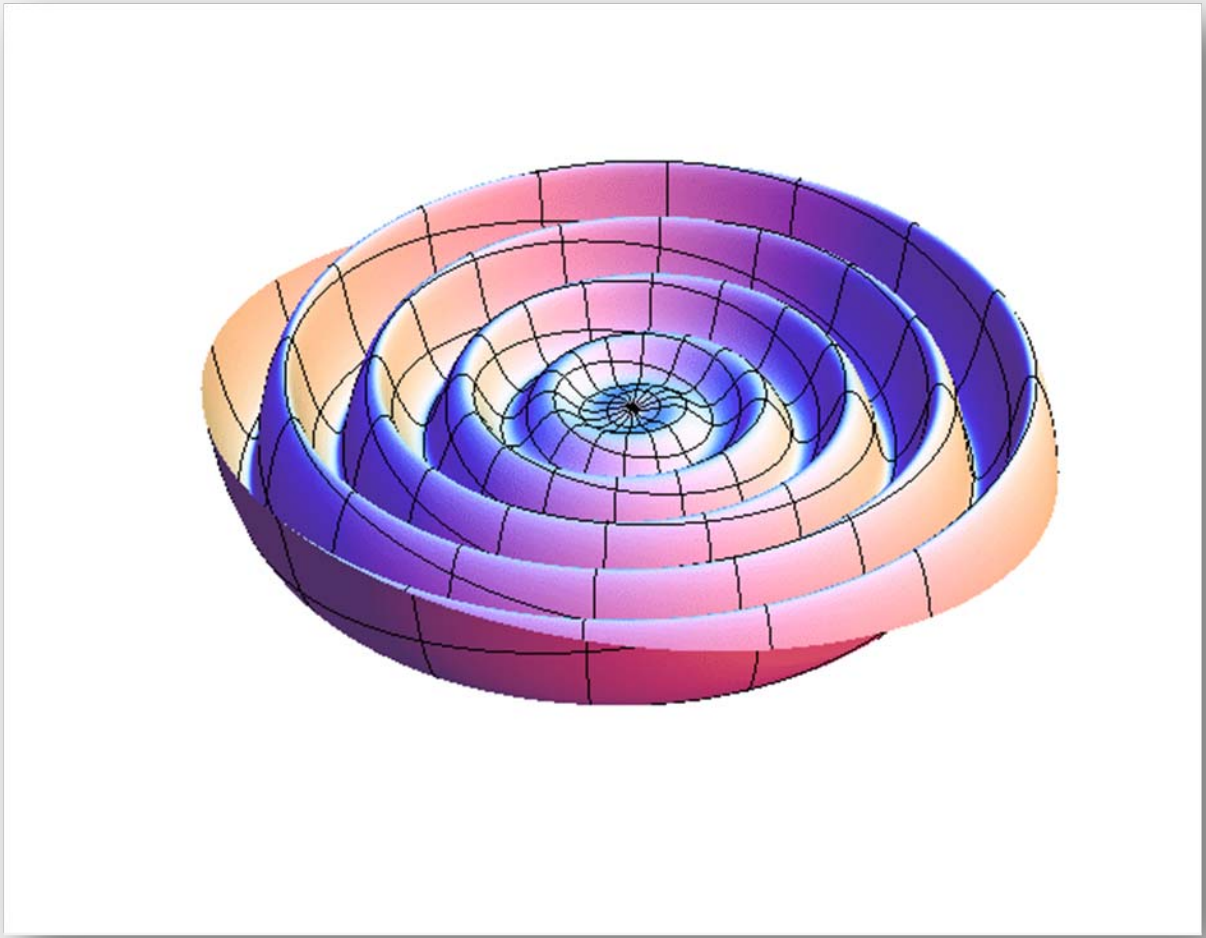
$$\theta_{ns} = \frac{\pi}{2} - \tan^{-1}\left(\tan\alpha \sin\phi^*\right); \quad \phi^* = \phi + \phi_0 + r\,\frac{\Omega}{V_{SW}}$$



- Neutral sheet drift a challenge going from 2D to 3D
- Consider magnetic field

$$\mathbf{B} = \mathbf{B}_m \left[1 - 2\mathrm{H}(\theta - \theta_{\mathrm{ns}})\right]$$

- Drift velocity for nearly-isotropic distribution – divergence-free

$$\mathbf{v}_D = \nabla \times \kappa_A \mathbf{e}_B$$

$$= \left(\nabla \times \kappa_A \frac{\mathbf{B}_m}{B_m} \right) \left[1 - 2\mathrm{H}(\theta - \theta_{\mathrm{ns}})\right]$$

$$+ 2\delta(\theta - \theta_{\mathrm{ns}}) \kappa_A \frac{\mathbf{B}_m}{B_m} \times \nabla(\theta - \theta_{\mathrm{ns}})$$

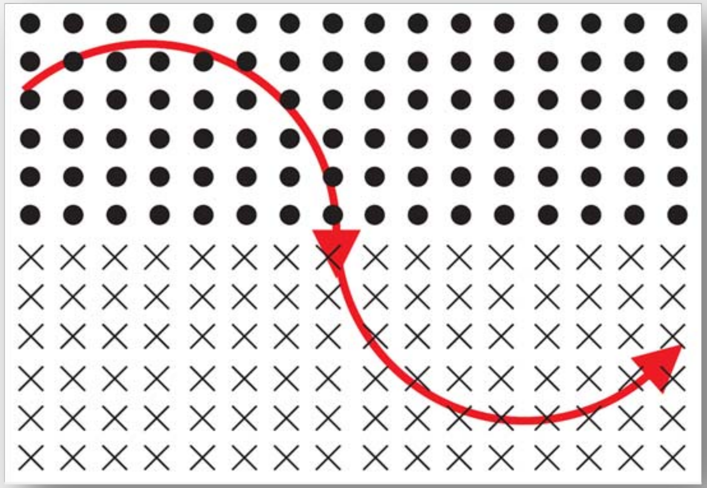
Result for a flat neutral sheet

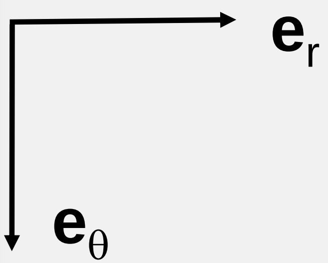
- For a flat sheet and purely azimuthal field the neutral sheet drift term in absence of scattering is

$$\mathbf{v}_{Dns} = 2\delta(\theta - \tfrac{\pi}{2})\kappa_A \mathbf{e}_{BM} \times \nabla(\theta - \tfrac{\pi}{2})$$

$$= -2\delta(\theta - \tfrac{\pi}{2})\frac{\upsilon R_L}{3}\mathbf{e}_\phi \times \left(\frac{1}{r}\mathbf{e}_\theta\right)$$

$$= 4R_L\delta(\theta - \tfrac{\pi}{2})\frac{\upsilon}{6}\frac{\mathbf{e}_r}{r}$$





Result for a flat neutral sheet

- Spatial average with $\Delta\theta_{ns} = 2R_L/r$ yields; verified with trajectory tracing (Burger et al. 1985, AstroSpSc)

$$\left\langle \mathbf{v}_{Dns} \right\rangle_\theta = \frac{1}{4R_L} \int_{\frac{\pi}{2}-\Delta\theta_{ns}}^{\frac{\pi}{2}+\Delta\theta_{ns}} 4R_L \delta(\theta - \tfrac{\pi}{2}) \frac{\upsilon}{6} \frac{\mathbf{e}_r}{r} \, r \, d\theta = \frac{\upsilon}{6} \mathbf{e}_r$$

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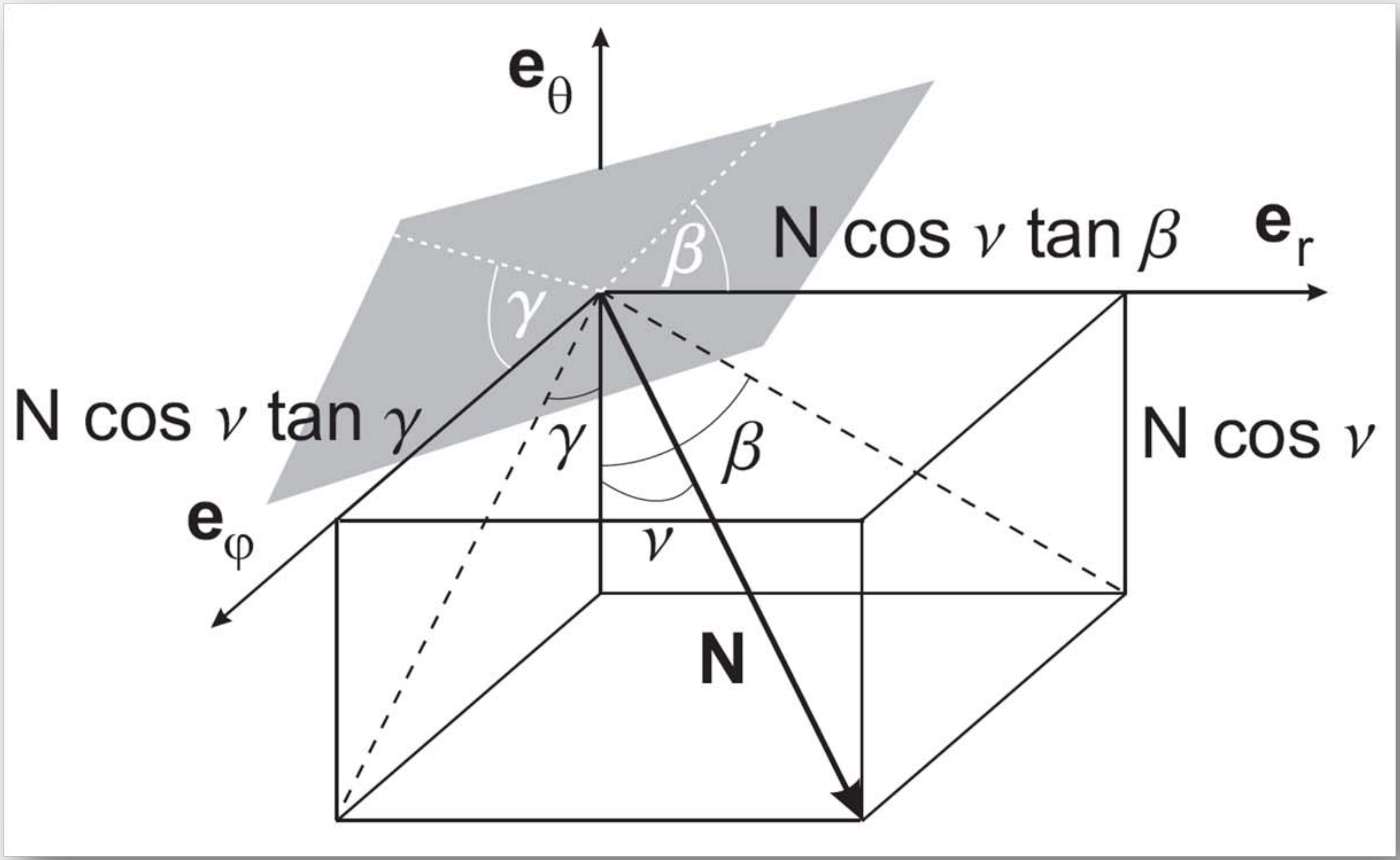
- Replace κ_A with κ_A cos ν tanh $[k(\theta_{ns}-\theta)]$ instead of using Heaviside function (ν defined on next slide; equal to 0° for a flat sheet)

$$\mathbf{v}_D = \left(\nabla \times \cos\nu \, \kappa_A \frac{\mathbf{B}_m}{B_m} \right) \tanh\left[k\left(\theta_{ns} - \theta\right)\right]$$

$$- \frac{k}{\cosh^2\left[k\left(\theta_{ns} - \theta\right)\right]} \kappa_A \frac{\mathbf{B}_m}{B_m} \times \nabla\left(\theta_{ns} - \theta\right)$$

Wavy neutral sheet – 3D geometry



Wavy neutral sheet – 3D geometry

- Angles γ and β are related to the spiral angle Ψ for Parker field …

$$\frac{\tan\beta}{\tan\gamma} = r\sin\theta\frac{\Omega}{V_{SW}} \equiv \tan\Psi$$

- …and to the angle ν between the normal and the meridional direction

$$\cos\nu = \frac{1}{\sqrt{1+\tan^2\beta+\tan^2\gamma}}$$

3D drift velocity field with a wavy neutral sheet

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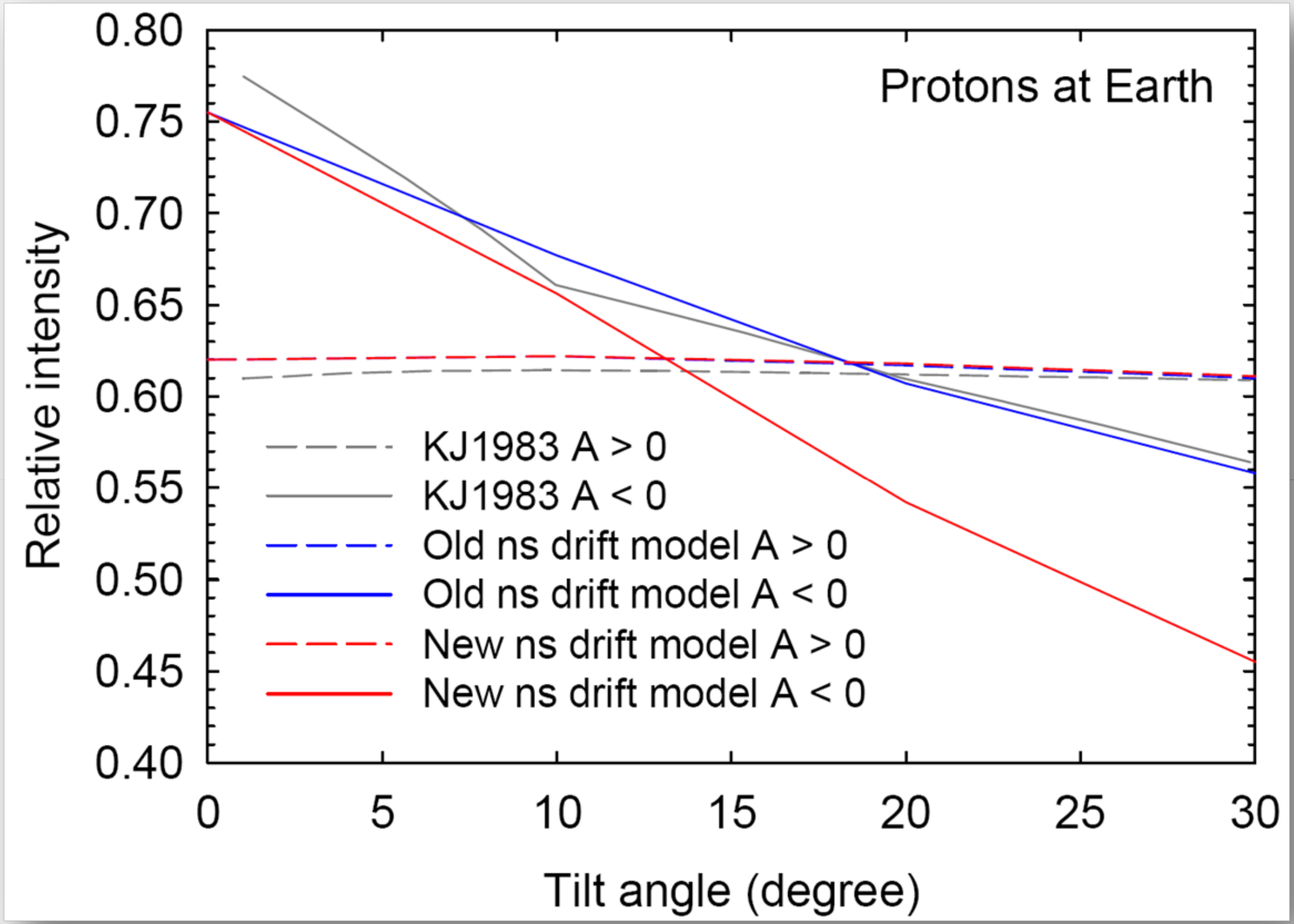
$$\mathbf{v}_D = \nabla \times \left(\kappa_A^e \mathbf{e}_B \right)$$

$$= \left(\nabla \times \kappa_A \cos\nu \frac{\mathbf{B}_m}{B_m} \right) \tanh\left[k \left(\theta_{ns} - \theta \right) \right]$$

$$+ \frac{k\kappa_A}{r} \frac{\left(\sin\Psi \cos\nu \, \mathbf{e}_r + \sin\nu \, \mathbf{e}_\theta + \cos\Psi \cos\nu \, \mathbf{e}_\phi \right)}{\cosh^2\left[k \left(\theta_{ns} - \theta \right) \right]}$$

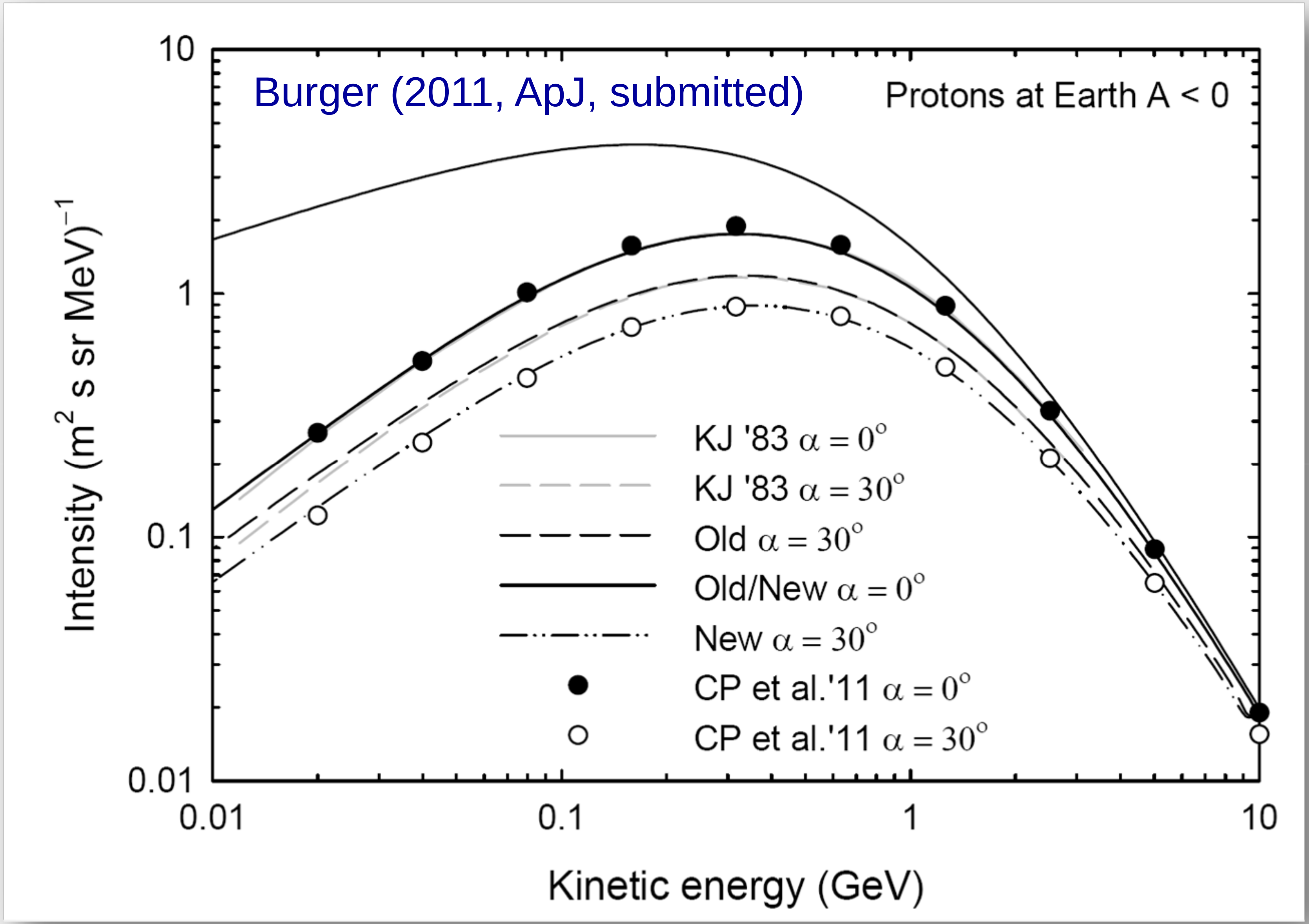
- Old approach without $\cos\nu$ results in neutral sheet drift speed that becomes unphysically large when tilt angle becomes too large – numerical model becomes unstable

Comparison with Kóta & Jokipii (1983, ApJ)



Burger (2011, ApJ, submitted)

Comparison with Kóta & Jokipii (1983, ApJ); Pei et al. (2011, ApJ)



7. Summary and conclusions

- As our understanding of modulation grows, so does the number of new questions waiting to be addressed
- Knowledge of turbulence spectra throughout heliosphere now a key input for a modulation model
- Drift coefficient requires more attention – theory and direct numerical simulations
- Drift along a wavy neutral sheet a key challenge for any fully 3D numerical modulation model – drift field for nearly-isotropic particle distribution must be locally and globally divergence-free