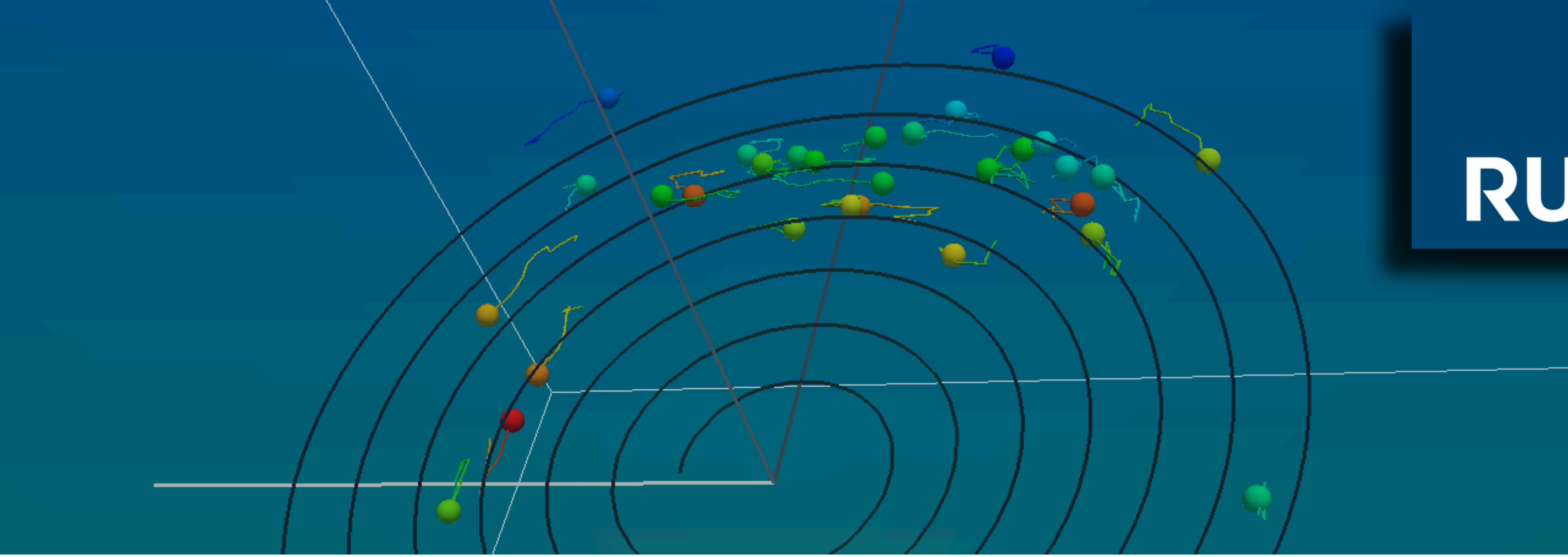

# A new model for fully anisotropic diffusion of CR protons and electrons in the heliosphere

## Cosmic Rays and the Heliospheric Plasma Environment, Bochum, 15.09.2011


***Frederic Effenberger,*** **Stephan Barra, Horst Fichtner, Jens Kleimann, DuToit Strauss and Klaus Scherer**

**Ruhr-Universität Bochum, FAKULTÄT FÜR PHYSIK**
Theoretische Physik IV - Weltraum und Astrophysik

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# Anisotropic Diffusion of Energetic Particles in Heliospheric and Galactic Magnetic Fields

Two faces of the same coin:

**Cosmic Ray modulation in the Heliosphere**

↕

**Cosmic Ray transport in the Galaxy**

$$\frac{\partial f}{\partial t} = \nabla \cdot (\hat{\kappa}\,\nabla f) - \vec{v} \cdot \nabla f + p\left(\frac{1}{3}(\nabla \cdot \vec{v}) + a_\pi\right)\frac{\partial f}{\partial p} + 3a_\pi f + \frac{q}{p^2}$$

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# Anisotropic Diffusion of Energetic Particles in Heliospheric and Galactic Magnetic Fields

**Cosmic Ray modulation in the Heliosphere**

**Cosmic Ray transport in the Galaxy**

$$\frac{\partial f}{\partial t} = \nabla \cdot (\hat{\kappa} \nabla f) - \vec{v} \cdot \nabla f + p \left( \frac{1}{3} (\nabla \cdot \vec{v}) + a_\pi \right) \frac{\partial f}{\partial p} + 3 a_\pi f + \frac{q}{p^2}$$

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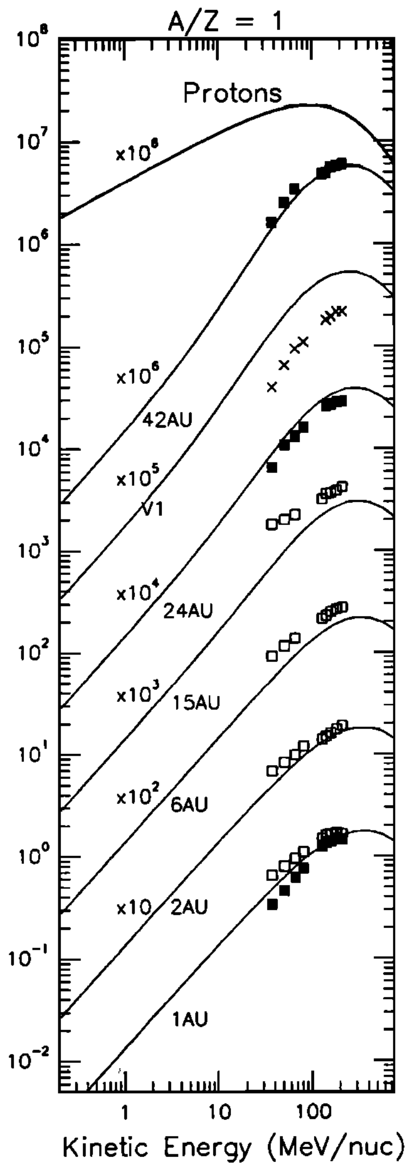
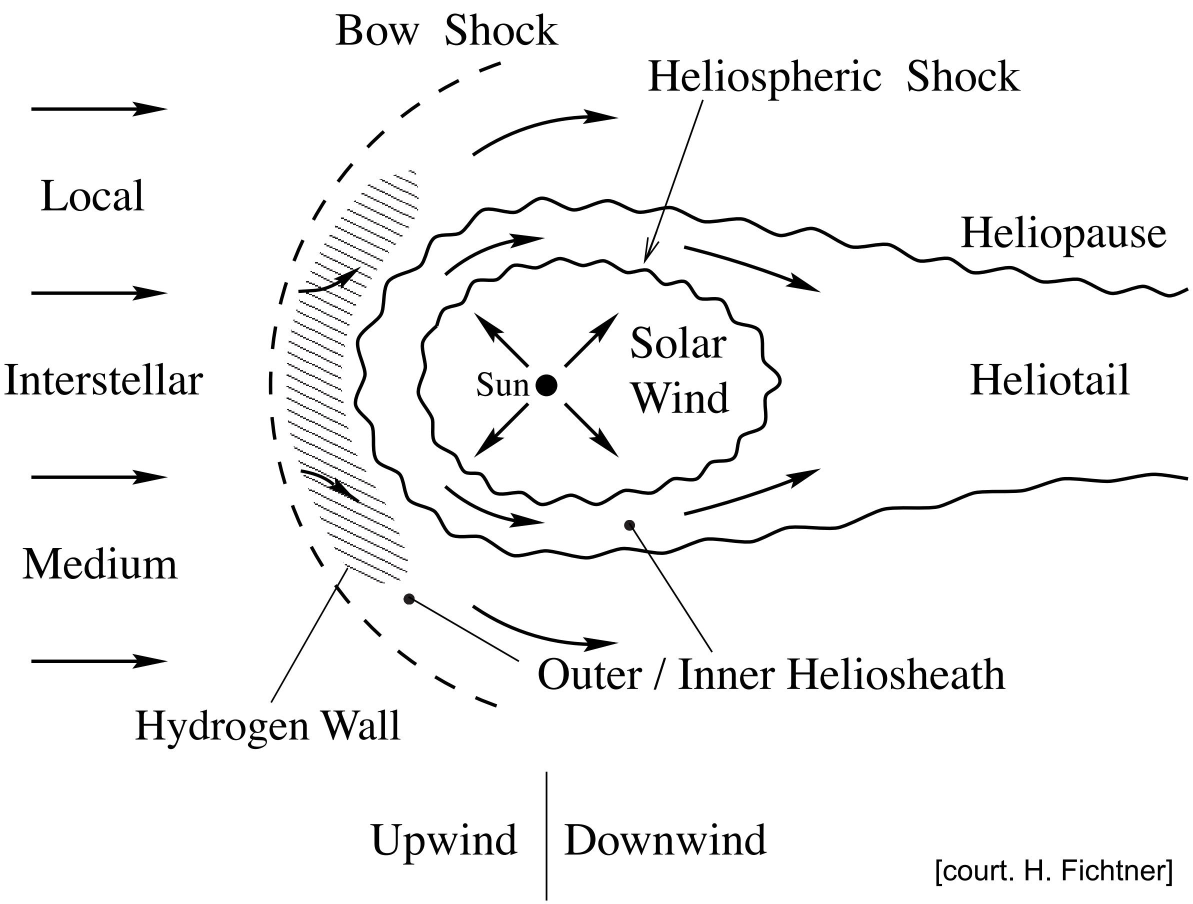
# Galactic Protons




[Reinecke et al., 1993, JGR]

$$j = 12.14 \cdot \beta(E_k + 0.5E_0)^{-2.6}$$




[court. H. Fichtner]

# Jovian Electrons

GEOPHYSICAL RESEARCH LETTERS, VOL. 27, NO. 11, PAGES 1611-1614, JUNE 1, 2000

**On the propagation of Jovian electrons in the heliosphere: transport modelling in 4-D phase space**

Horst Fichtner

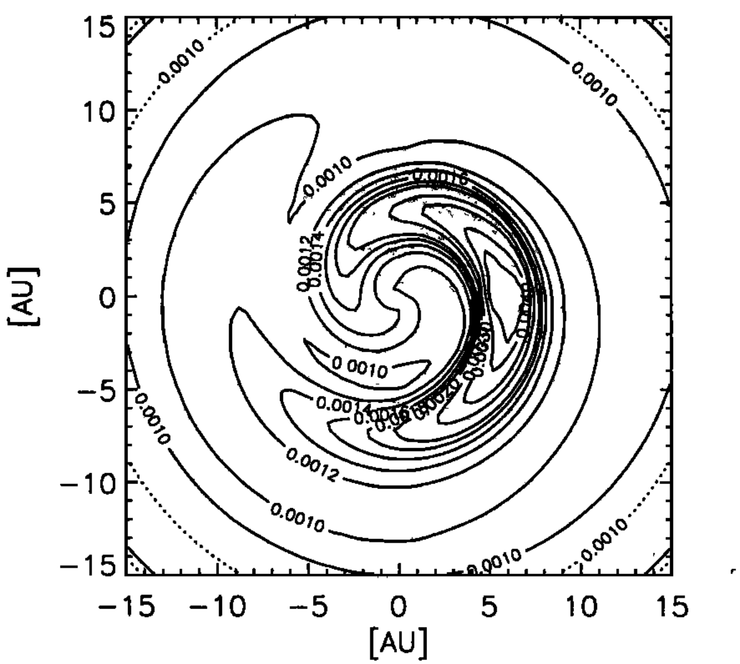
Institut für Theoretische Physik IV: Weltraum- und Astrophysik, Ruhr-Univ. Bochum, Germany

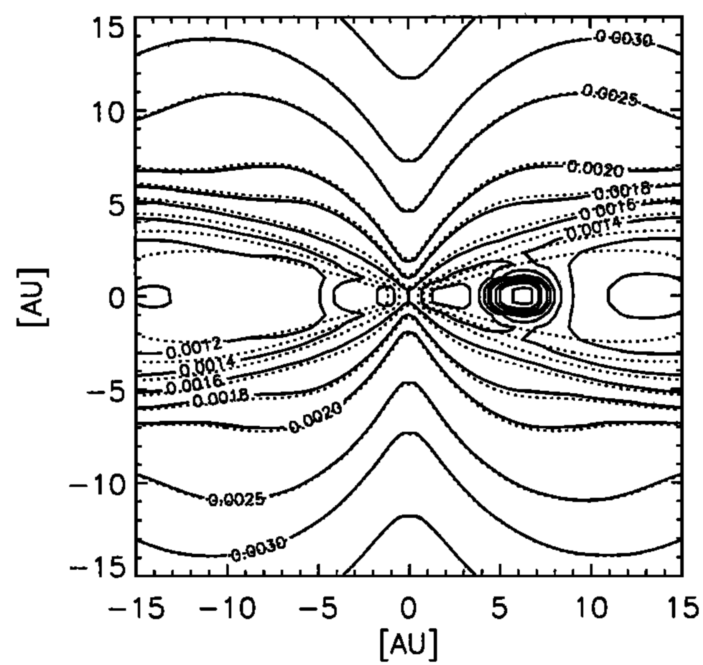
Marius Potgieter, Stefan Ferreira, and Adri Burger

Space Res. Unit, School of Physics, Potchefstroom Univ. for CHE, Potchefstroom, South Africa

$$j_{gal}(R(p)) = \delta\, exp\left(\alpha(ln\, R)^2 + \beta\, ln\, R + \gamma\right) \quad (5)$$

$$j_{Jov}(E(p)) = j(E_r)\,\frac{E_r^{1.5}\,(h+E_r)^{3.5}}{E^{1.5}\,(h+E)^{3.5}} \quad (6)$$



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# Two Questions,
# Two Complementary Numerical Tools

Heliophysical Questions:

- How are the cosmic ray protons and electrons modulated on their way from the heliospheric boundary to earth?
- Can we determine transport parameters from the propagation of Jovian electrons?

---

- Grid-Based Numerics: DuFort-Frankel, VLUGR3 ...

Solving the CR-Transport Equation directly on a numerical grid via finite differences or finite volume methods.

- Stochastic Differential Equations (SDE)

Solve a SDE equivalent to the Transport Equation by tracing pseudo-particles

# Testing the SDE Modulation with an Analytic Parker Propagator

## The Parker propagator for spherical solar modulation

**O. Stawicki, H. Fichtner, and R. Schlickeiser**

Institut für Theoretische Physik, Lehrstuhl IV: Weltraum- und Astrophysik, Ruhr-Universität Bochum, 44780 Bochum, Germany

Received 28 January 2000 / Accepted 30 March 2000

$$
\begin{aligned}
j(r,p) &= p^2 f(r,p) \\
&= \frac{3}{b_p} \int dr_0 \int dp_0 \frac{S(r_0,p_0)}{V(r_0)} \frac{p_0 y_0}{f_p} \left(\frac{r_0}{r}\right)^{\frac{1+\beta_p}{2}} \\
&\times \left(\frac{p_0}{p}\right)^{\frac{3\beta_p - 4\alpha_p - 5}{2(2+\alpha_p)}} \exp\left(-\frac{y_0(1+h_p^2)}{f_p}\right) \\
&\times I_{\frac{1+\beta_p}{1+\alpha_p-\beta_p}}\left(\frac{2y_0 h_p}{f_p}\right)
\end{aligned}
$$

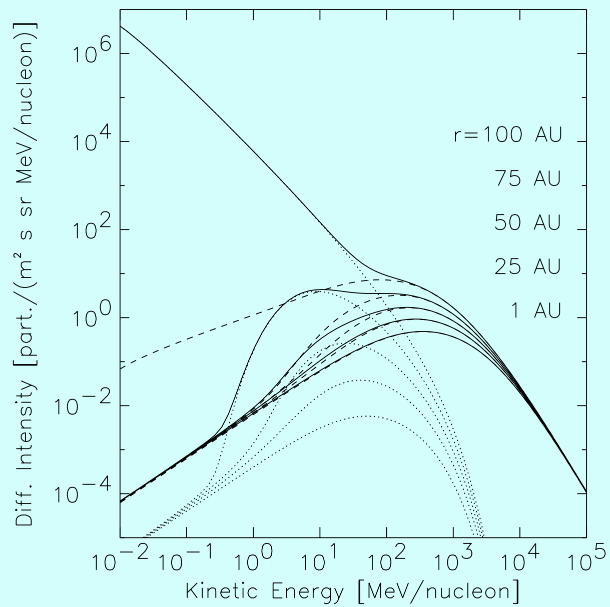


**Fig. 1.** The modulated spectra of ACRs and GCRs in the heliosphere. The solar wind termination shock marking the position of the sources is located at $r_{\rm sh} = 100$ AU. The solid lines are the combined spectra, the dotted and dashed lines indicate the individual contributions from ACRs and GCRs, respectively.

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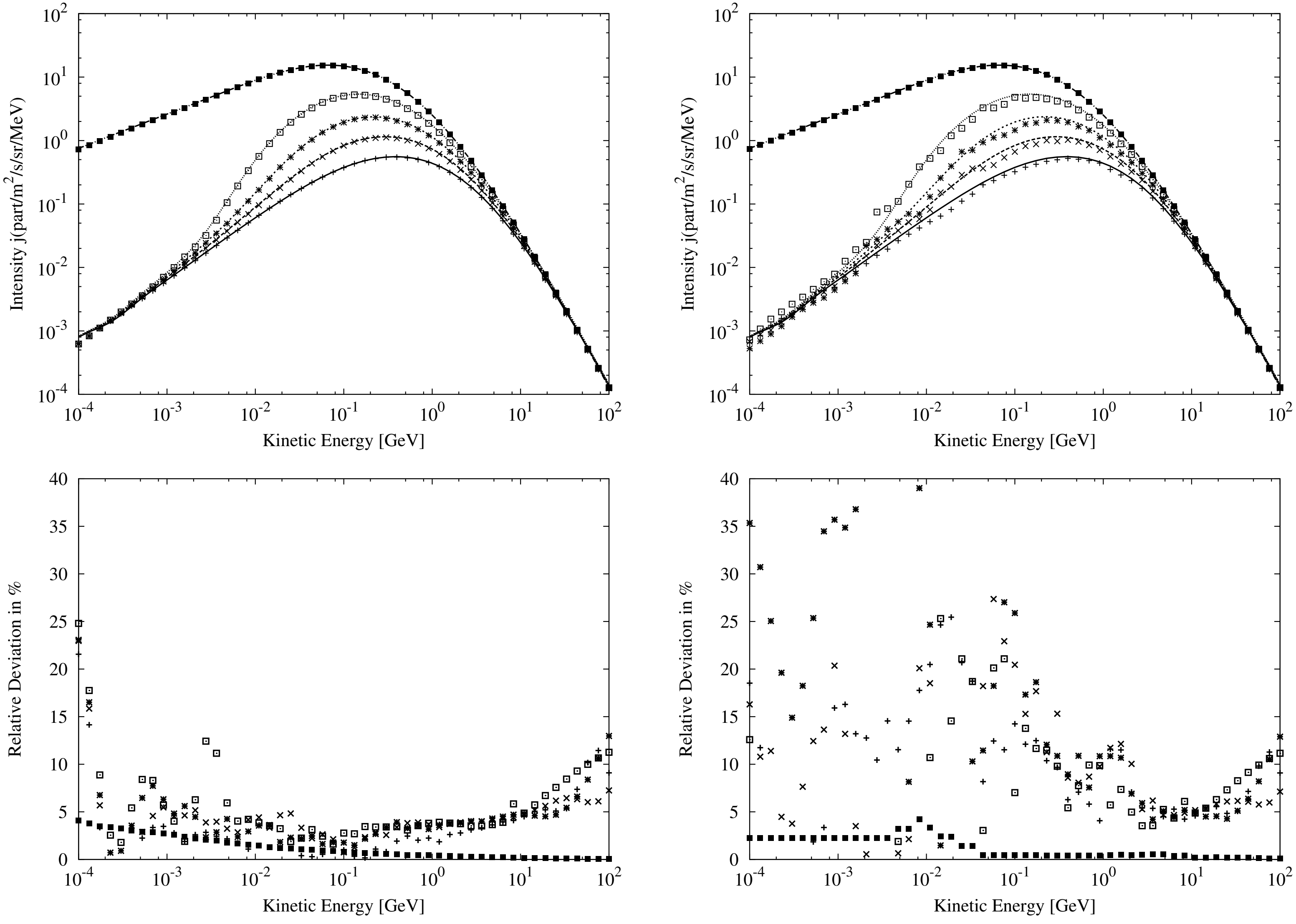
# Results

## Testing the SDE Modulation with an Analytic Parker Propagator




10000 particles

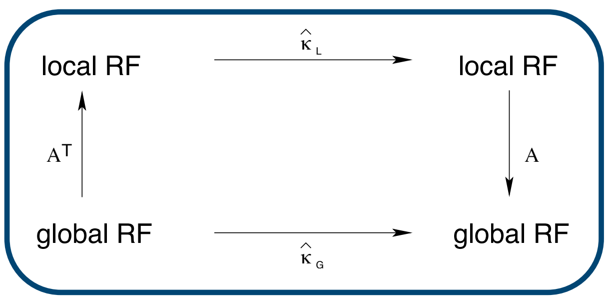
100 particles

# Fully Anisotropic Diffusion

$$\frac{\partial f}{\partial t} = \nabla \cdot (\hat{\kappa} \nabla f)$$

local Tensor

$$\hat{\kappa}_L = \begin{pmatrix} \kappa_{\perp 1} & 0 & 0 \\ 0 & \kappa_{\perp 2} & 0 \\ 0 & 0 & \kappa_{||} \end{pmatrix}$$



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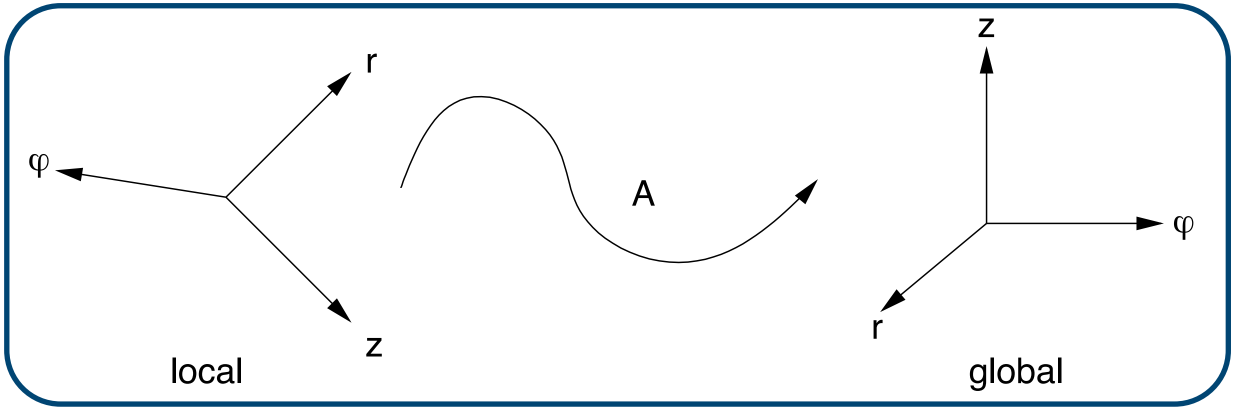


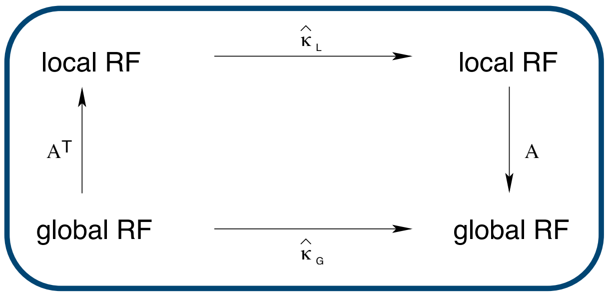

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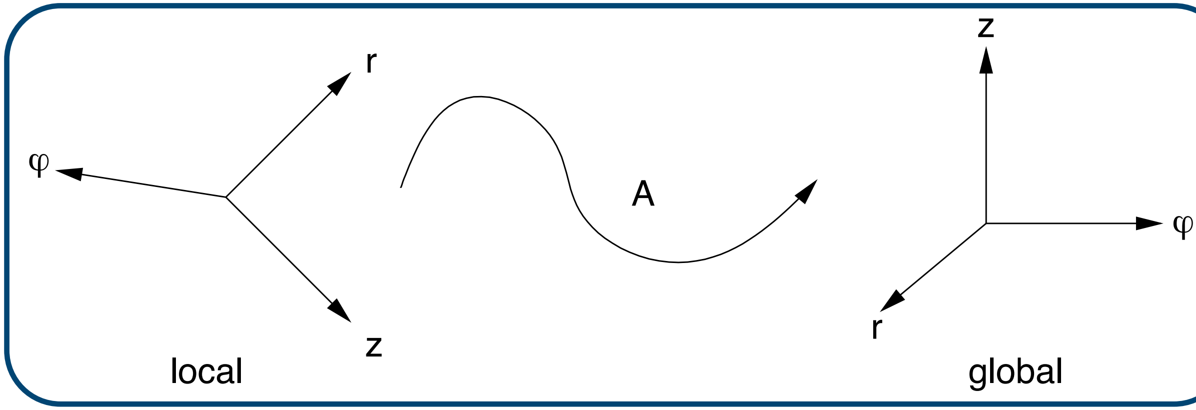
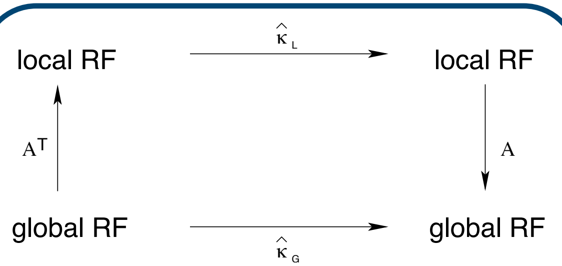


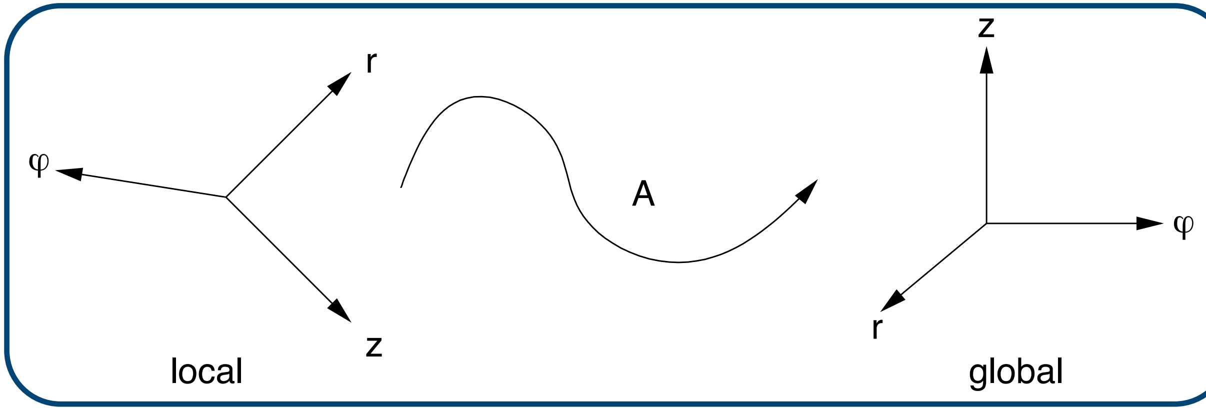

$$\hat{\kappa}_G = A^{-1} \hat{\kappa}_L A = A^T \hat{\kappa}_L A$$

# Fully Anisotropic Diffusion

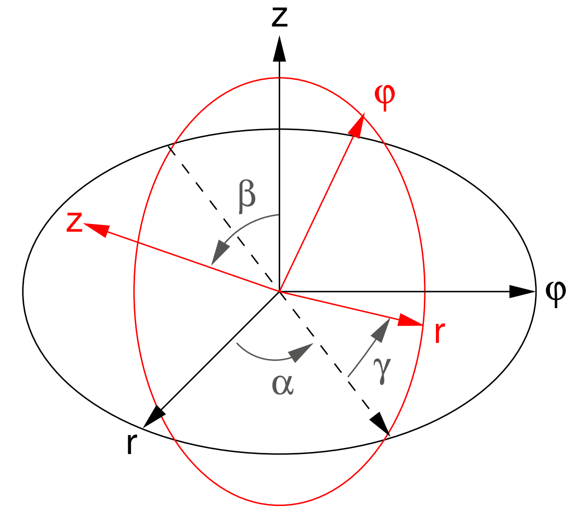
$$\frac{\partial f}{\partial t} = \nabla \cdot (\hat{\kappa}\, \nabla f)$$

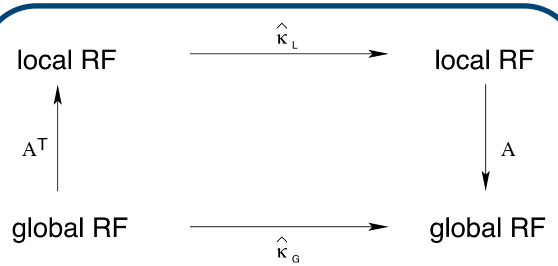
local Tensor

$$\hat{\kappa}_L = \begin{pmatrix} \kappa_{\perp 1} & 0 & 0 \\ 0 & \kappa_{\perp 2} & 0 \\ 0 & 0 & \kappa_{\|} \end{pmatrix}$$




Euler-Angle Transformation:




$$\hat{\kappa}_G = A^{-1}\hat{\kappa}_L A = A^T \hat{\kappa}_L A$$

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# Anisotropic Diffusion

## The „Classical" Approach: Euler-Angle Transformation

$$
\begin{aligned}
\kappa_{rr} &= \kappa_{\perp,2} \sin^2\zeta + \cos^2\zeta\big(\kappa_{||} \cos^2\Psi + \kappa_{\perp,3} \sin^2\Psi\big), \\
\kappa_{r\theta} &= -\kappa_{\mathrm{A}} \sin\Psi + \sin\zeta \cos\zeta \\
&\quad \times \big(\kappa_{||} \cos^2\Psi + \kappa_{\perp,3} \sin^2\Psi - \kappa_{\perp,2}\big), \\
\kappa_{r\phi} &= -\kappa_{\mathrm{A}} \cos\Psi \sin\zeta - \big(\kappa_{||} - \kappa_{\perp,3}\big) \sin\Psi \cos\Psi \cos\zeta, \\
\kappa_{\theta r} &= \kappa_{\mathrm{A}} \sin\Psi + \sin\zeta \cos\zeta \\
&\quad \times \big(\kappa_{||} \cos^2\Psi + \kappa_{\perp,3} \sin^2\Psi - \kappa_{\perp,2}\big), \\
\kappa_{\theta\theta} &= \kappa_{\perp,2} \cos^2\zeta + \sin^2\zeta\big(\kappa_{||} \cos^2\Psi + \kappa_{\perp,3} \sin^2\Psi\big), \\
\kappa_{\theta\phi} &= \kappa_{\mathrm{A}} \cos\Psi \cos\zeta - \big(\kappa_{||} - \kappa_{\perp,3}\big) \sin\Psi \cos\Psi \sin\zeta, \\
\kappa_{\phi r} &= \kappa_{\mathrm{A}} \cos\Psi \sin\zeta - \big(\kappa_{||} - \kappa_{\perp,3}\big) \sin\Psi \cos\Psi \cos\zeta, \\
\kappa_{\phi\theta} &= -\kappa_{\mathrm{A}} \cos\Psi \cos\zeta - \big(\kappa_{||} - \kappa_{\perp,3}\big) \sin\Psi \cos\Psi \sin\zeta, \\
\kappa_{\phi\phi} &= \kappa_{||} \sin^2\Psi + \kappa_{\perp,3} \cos^2\Psi, \qquad (17)
\end{aligned}
$$

where $\tan\Psi = -B_\phi/\big(B_r^2 + B_\theta^2\big)^{1/2}$ and $\tan\zeta = B_\theta/B_r$. Note that

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# Anisotropic Diffusion

## The new Approach: Local Frenet-Trihedron

Tangential Vector $\vec{t} = \frac{\vec{B}}{B}$

$$(\vec{t} \cdot \nabla)\vec{t} = k\vec{n} \; ; \; (\vec{t} \cdot \nabla)\vec{n} = -k\vec{t} + \tau\vec{b} \; ; \; (\vec{t} \cdot \nabla)\vec{b} = -\tau\vec{n}$$

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# Anisotropic Diffusion

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Normal Vector $\vec{n}$

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Normal Vector $\vec{n}$

Binormal Vector $\vec{b} = \vec{t} \times \vec{n}$

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# Anisotropic Diffusion

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Normal Vector $\vec{n}$

Binormal Vector $\vec{b} = \vec{t} \times \vec{n}$

$$A = \begin{pmatrix} n_1 & b_1 & t_1 \\ n_2 & b_2 & t_2 \\ n_3 & b_3 & t_3 \end{pmatrix}$$

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# Anisotropic Diffusion

## The General Transformation

The symmetric global diffusion tensor:

$$
\begin{aligned}
\hat{\kappa}_{11} &= \kappa_{\perp 1} n_1^2 + \kappa_{\perp 2} b_1^2 + \kappa_{\|} t_1^2 \\
\hat{\kappa}_{12} &= \kappa_{\perp 1} n_1 n_2 + \kappa_{\perp 2} b_1 b_2 + \kappa_{\|} t_1 t_2 \\
\hat{\kappa}_{13} &= \kappa_{\perp 1} n_1 n_3 + \kappa_{\perp 2} b_1 b_3 + \kappa_{\|} t_1 t_3 \\
\hat{\kappa}_{22} &= \kappa_{\perp 1} n_2^2 + \kappa_{\perp 2} b_2^2 + \kappa_{\|} t_2^2 \\
\hat{\kappa}_{23} &= \kappa_{\perp 1} n_2 n_3 + \kappa_{\perp 2} b_2 b_3 + \kappa_{\|} t_2 t_3 \\
\hat{\kappa}_{33} &= \kappa_{\perp 1} n_3^2 + \kappa_{\perp 2} b_3^2 + \kappa_{\|} t_3^2
\end{aligned}
$$

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\end{aligned}$$

General magnetic field &
local diffusion coefficients

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# Anisotropic Diffusion

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\end{aligned}
$$

General magnetic field &
local diffusion coefficients

↓

Calculate local trihedron

RUB

# Anisotropic Diffusion
## The General Transformation

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\end{aligned}
$$

General magnetic field & local diffusion coefficients

↓

Calculate local trihedron

↓

Include in transformation to global reference frame

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# Anisotropic Diffusion
## The General Transformation

The symmetric global diffusion tensor:

$$
\begin{aligned}
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\end{aligned}
$$

General magnetic field & local diffusion coefficients

↓

Calculate local trihedron

↓

Include in transformation to global reference frame

↓

General, fully anisotropic, global diffusion tensor

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# Anisotropic Diffusion

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\end{aligned}
$$

General magnetic field & local diffusion coefficients

↓

Calculate local trihedron

↓

Include in transformation to global reference frame

↓

General, fully anisotropic, global diffusion tensor

In any case: The simple structure of the diffusion tensor in the local frame may lead to complicated tensor elements in the global frame, depending on the magnetic field.

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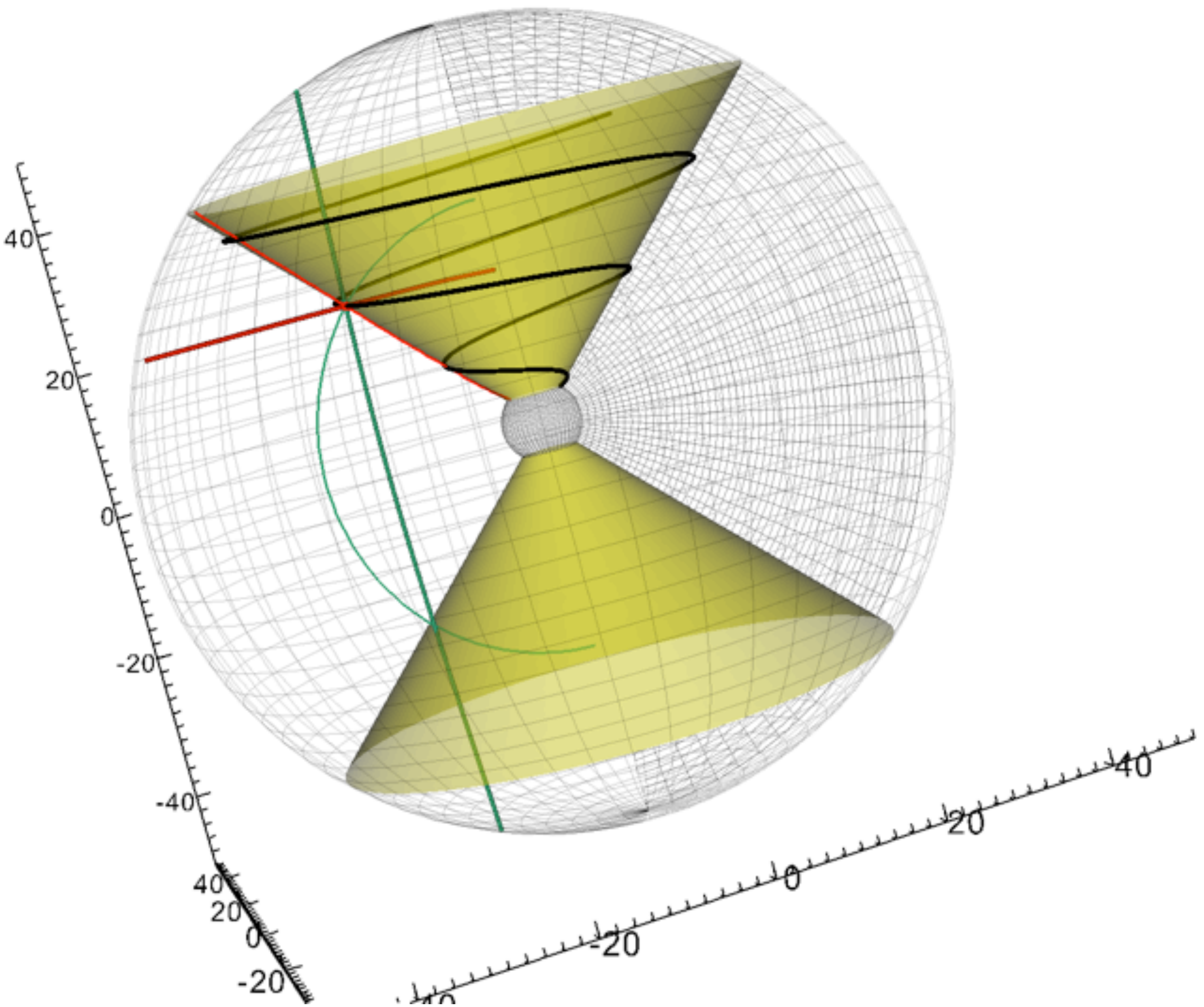
# Anisotropic Diffusion

## The Parker-Field Trihedron

$$B_\rho(r,\theta,\varphi) = B_0 \left(\frac{r_0}{r}\right)^2$$
$$B_\theta(r,\theta,\varphi) = 0$$
$$B_\varphi(r,\theta,\varphi) = -B_0 \left(\frac{r_0}{r}\right)^2 \left(\frac{\Omega_\odot (r - r_\odot)\sin\theta}{v_{sw}}\right)$$



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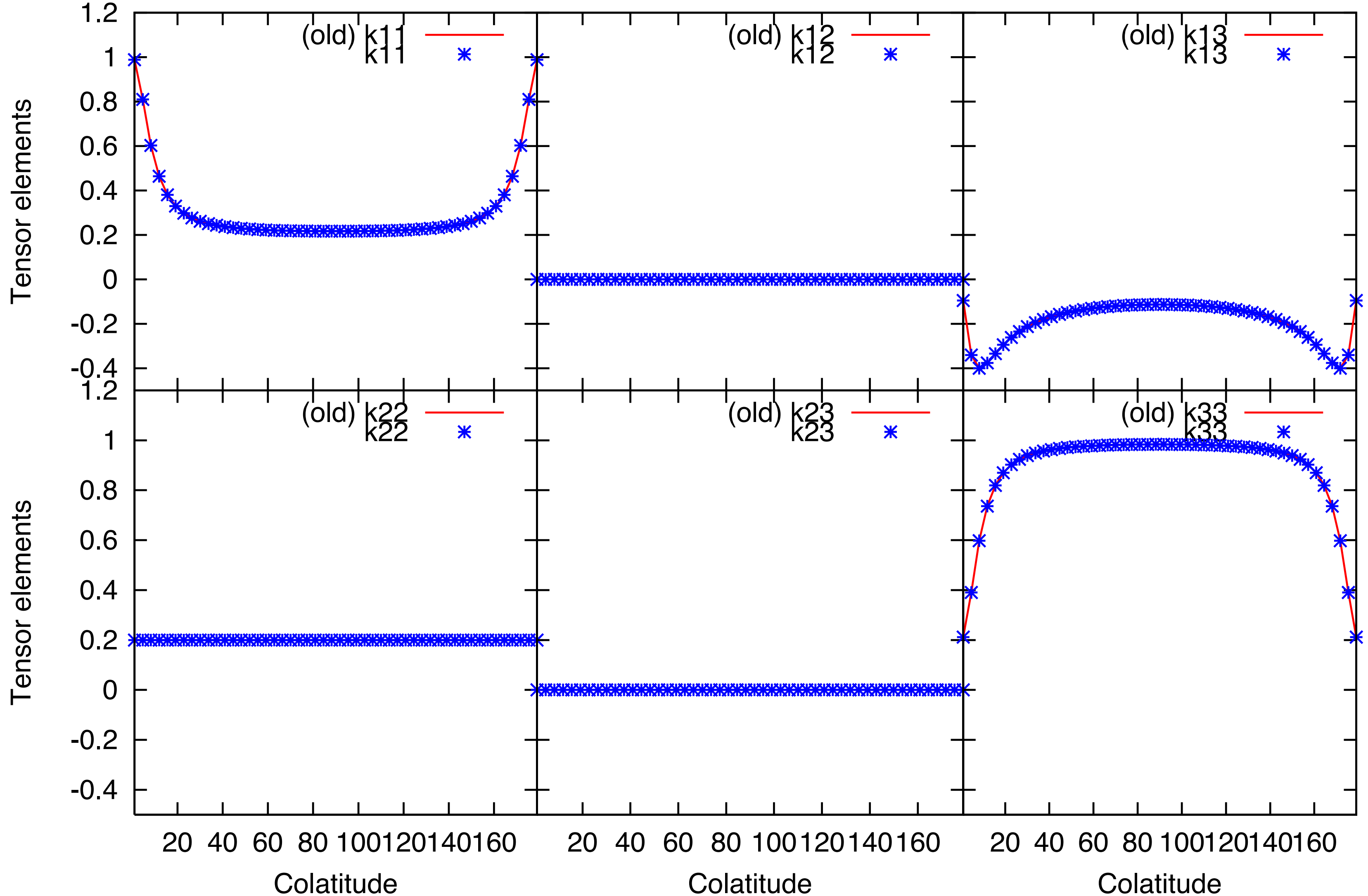
# Anisotropic Diffusion

## The Parker-Field Tensor Elements (isotropic perp.)

$\kappa_{\|} = 1,\ \kappa_{\perp 1} = 0.2,\ \kappa_{\perp 2} = 0.2$
$r = 10\mathrm{AU},\ \varphi = \pi$



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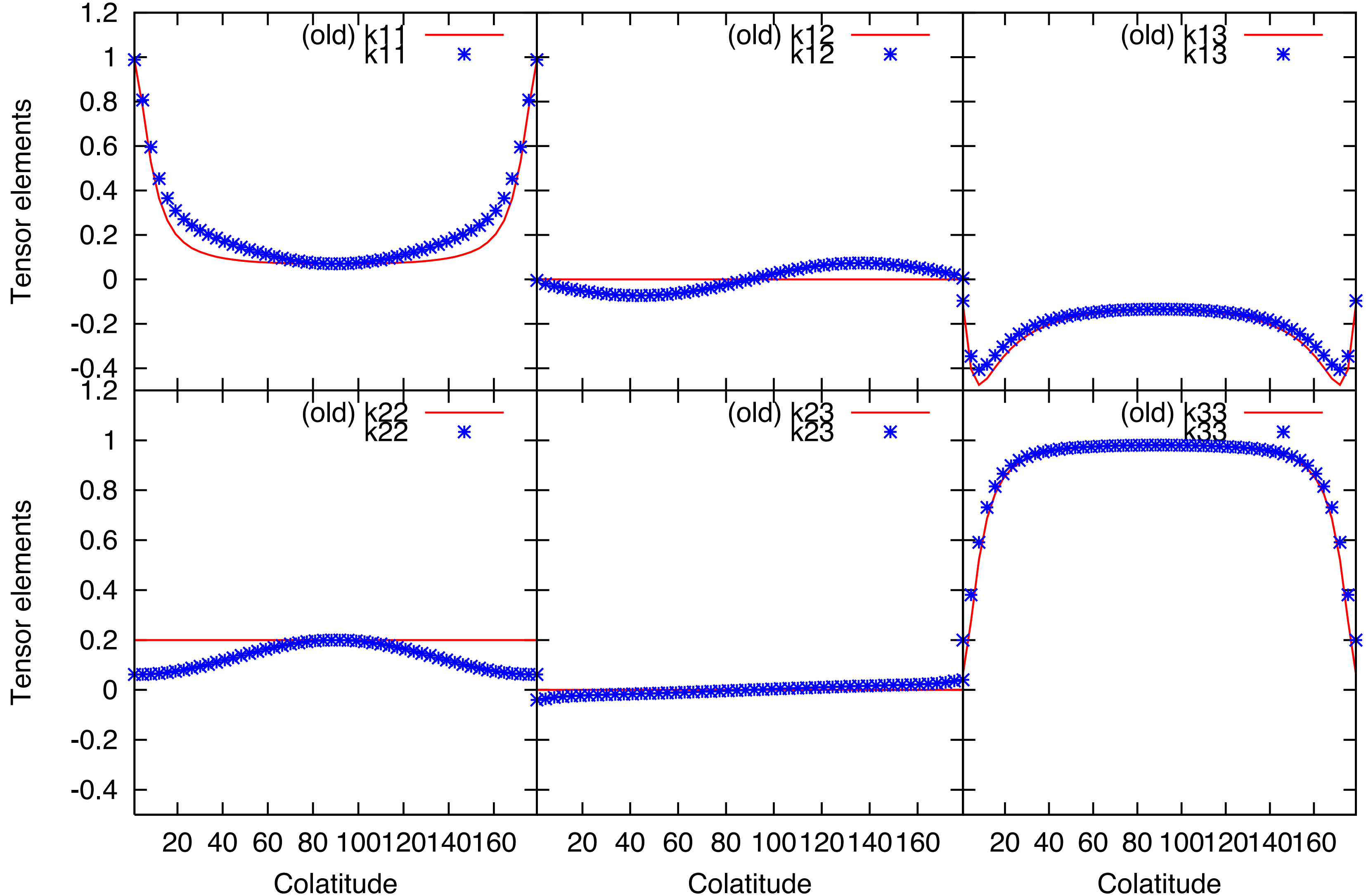
# Anisotropic Diffusion

## The Parker-Field Tensor Elements (anisotropic perp.)

$\kappa_{\|} = 1,\ \kappa_{\perp 1} = 0.05,\ \kappa_{\perp 2} = 0.2$
$r = 10\mathrm{AU},\ \varphi = \pi$



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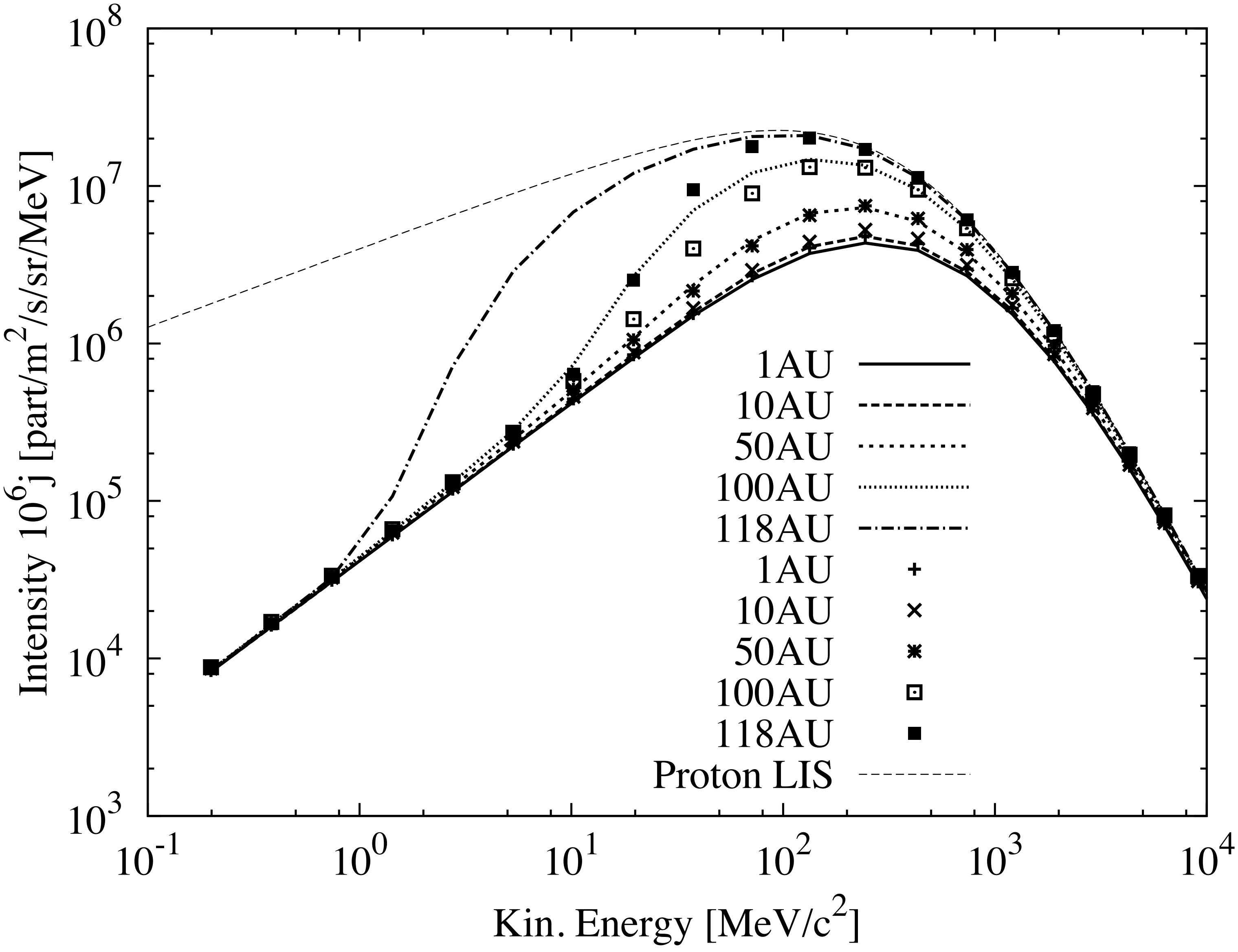
# Results

## Modulation Spectra Isotropic vs Anisotropic



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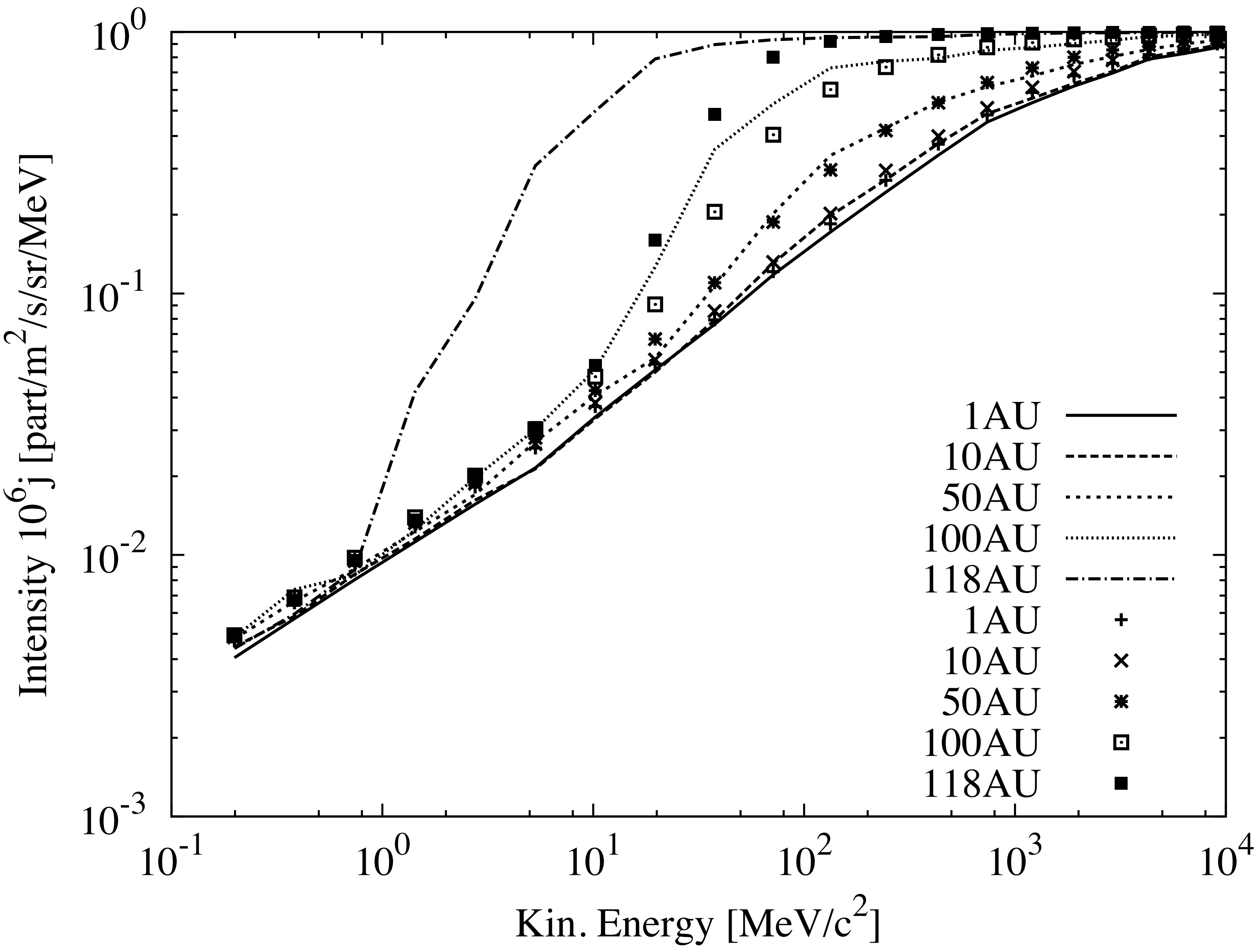
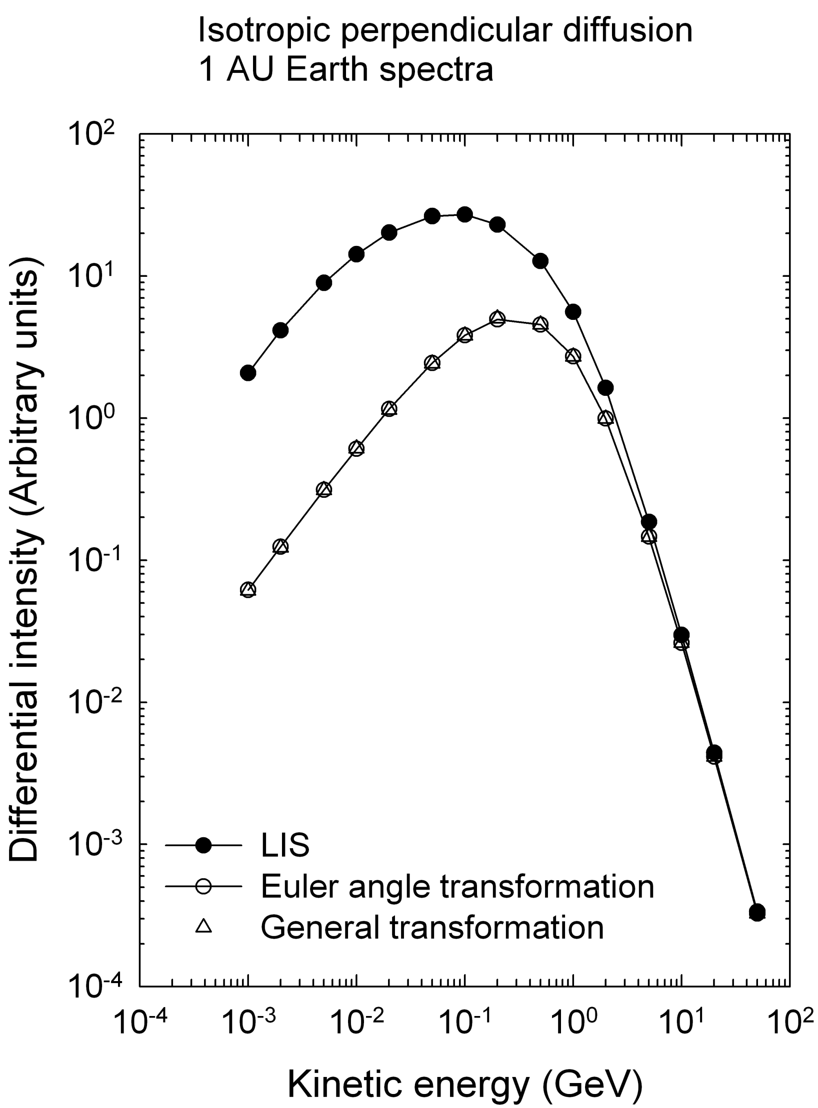
# Results

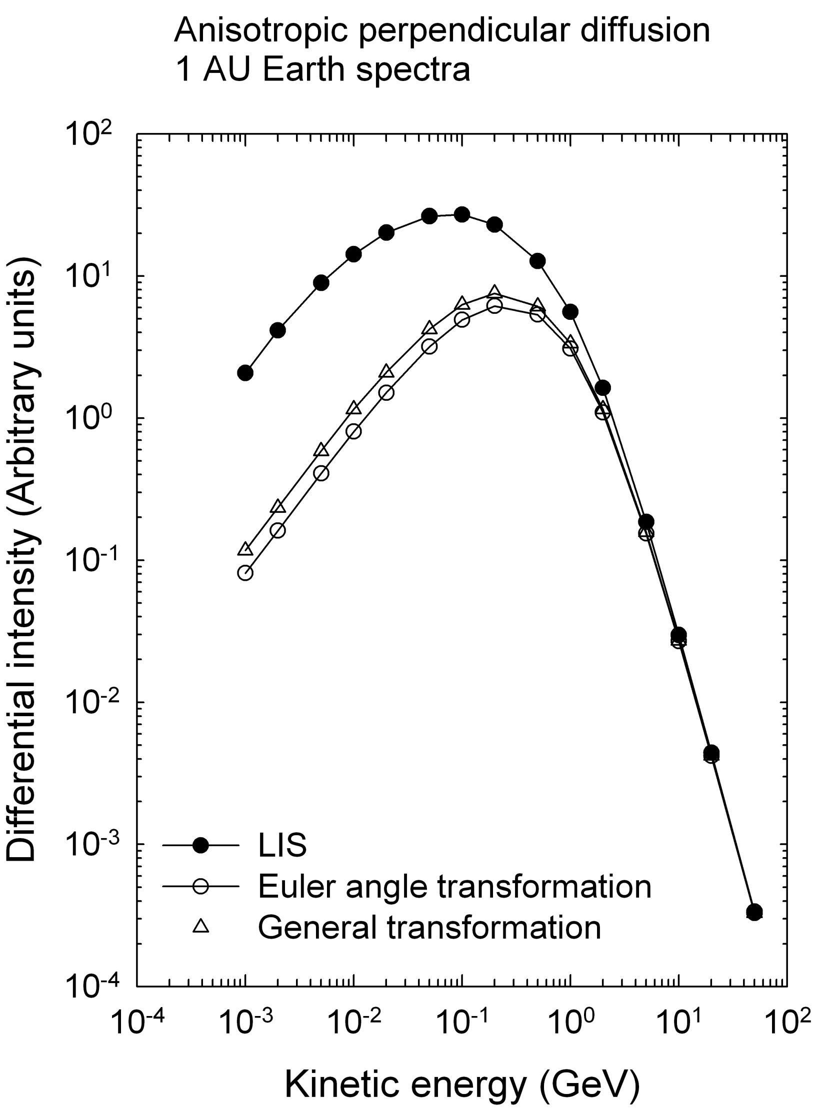
## Modulation Spectra Isotropic vs Anisotropic (Normalized to LIS)



# Results

## Effect of Different Tensor Formulations

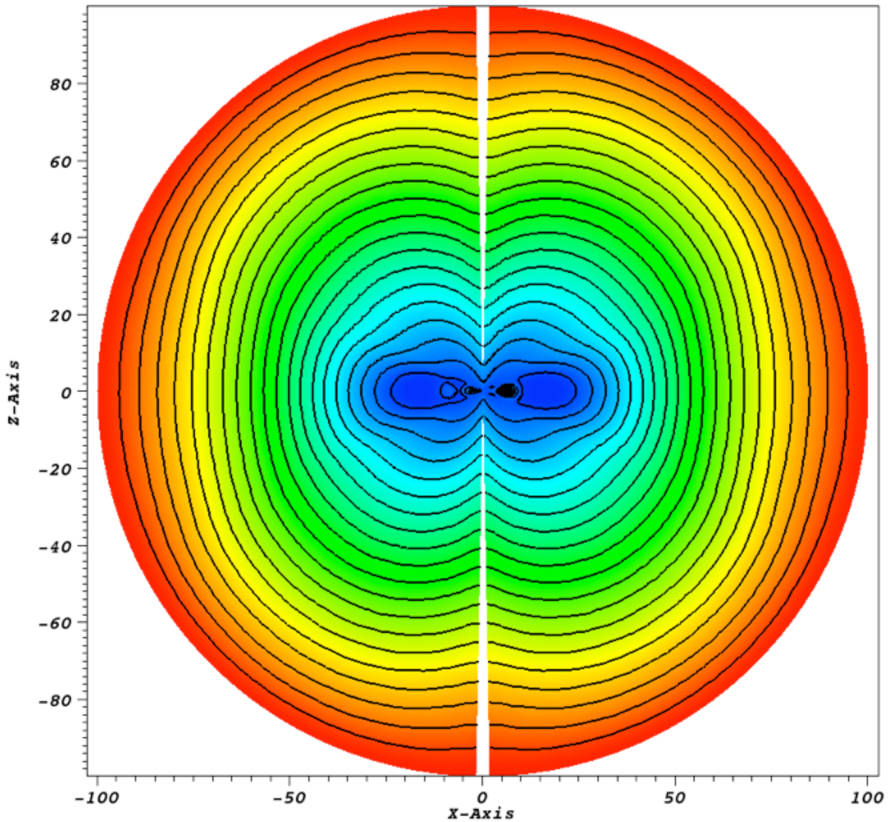


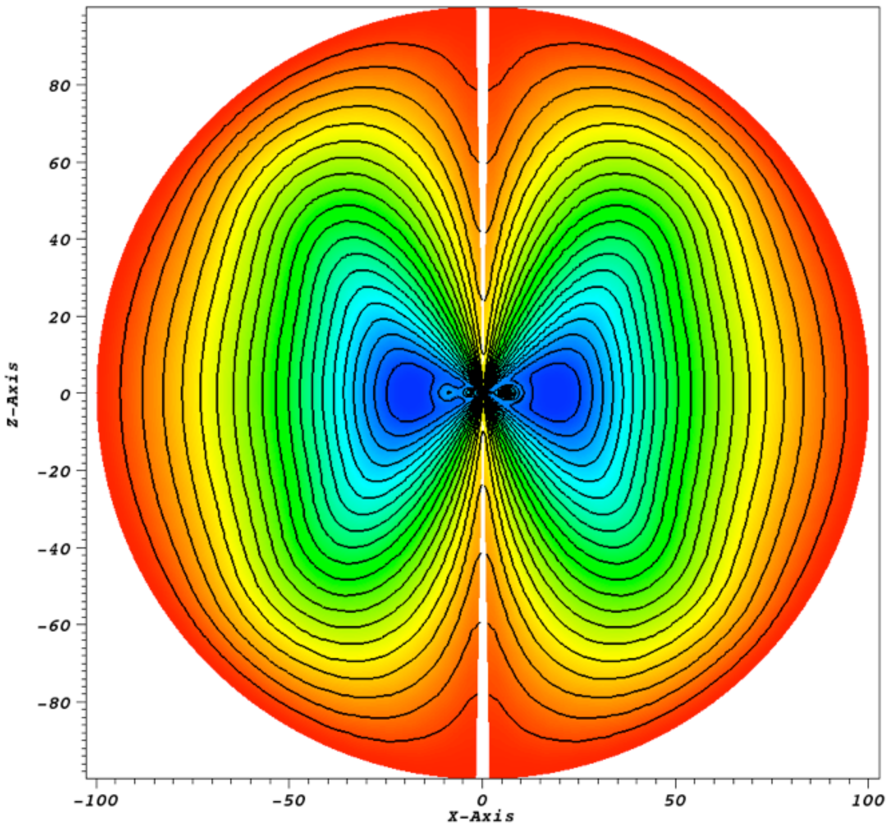




[court. D. Strauss]

# Results

## Jovian Electron Distribution (8 MeV)




With Standard Modification:

$$F(\theta) = \quad C^{+} + C^{-} \tanh\left[(\tilde{\theta} - 90^{\circ} - \theta_F)/\Delta\theta\right]$$

Isotropic Perpendicular Diffusion

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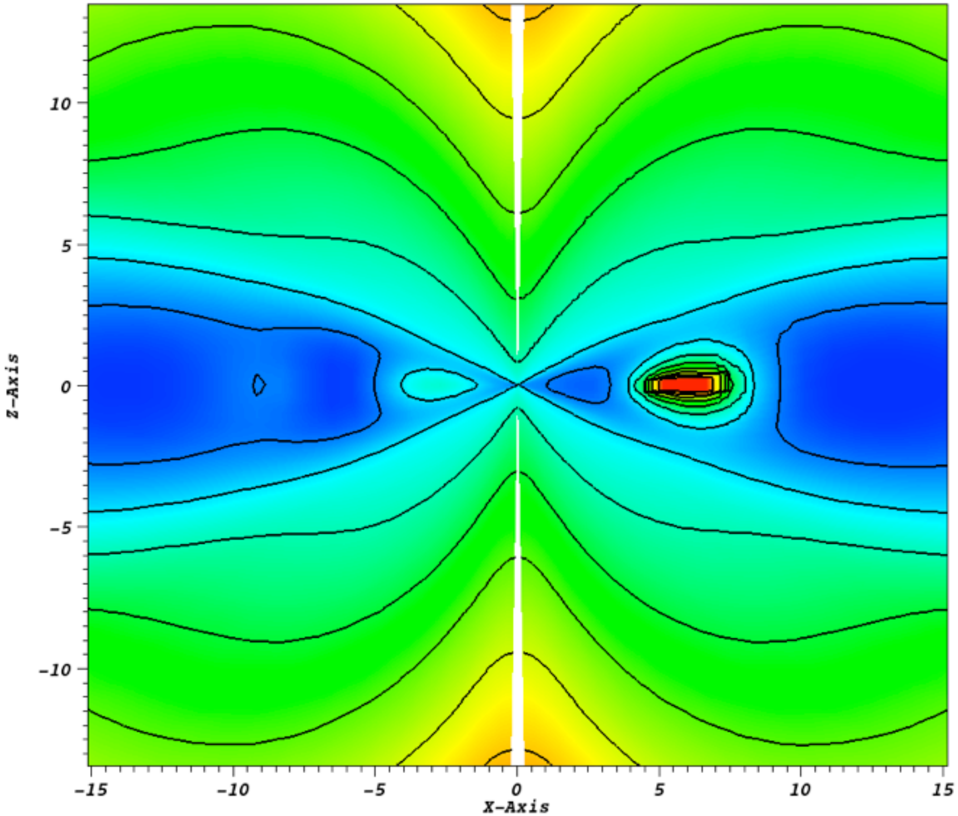
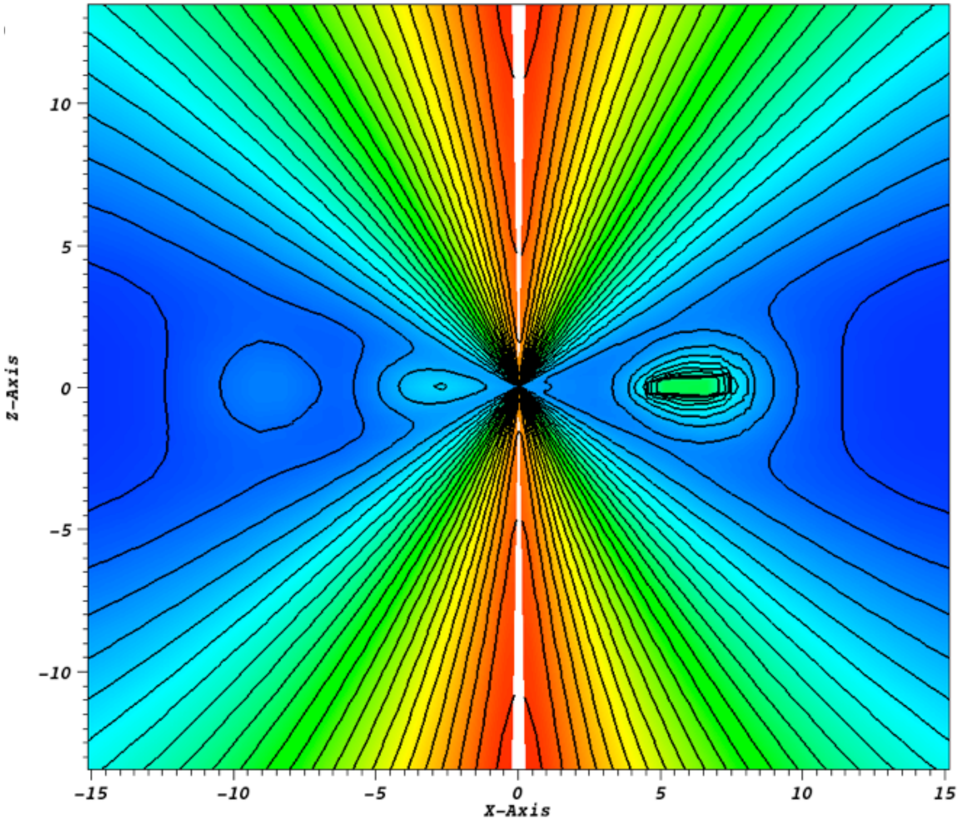
# Results

## Jovian Electron Distribution (8 MeV)



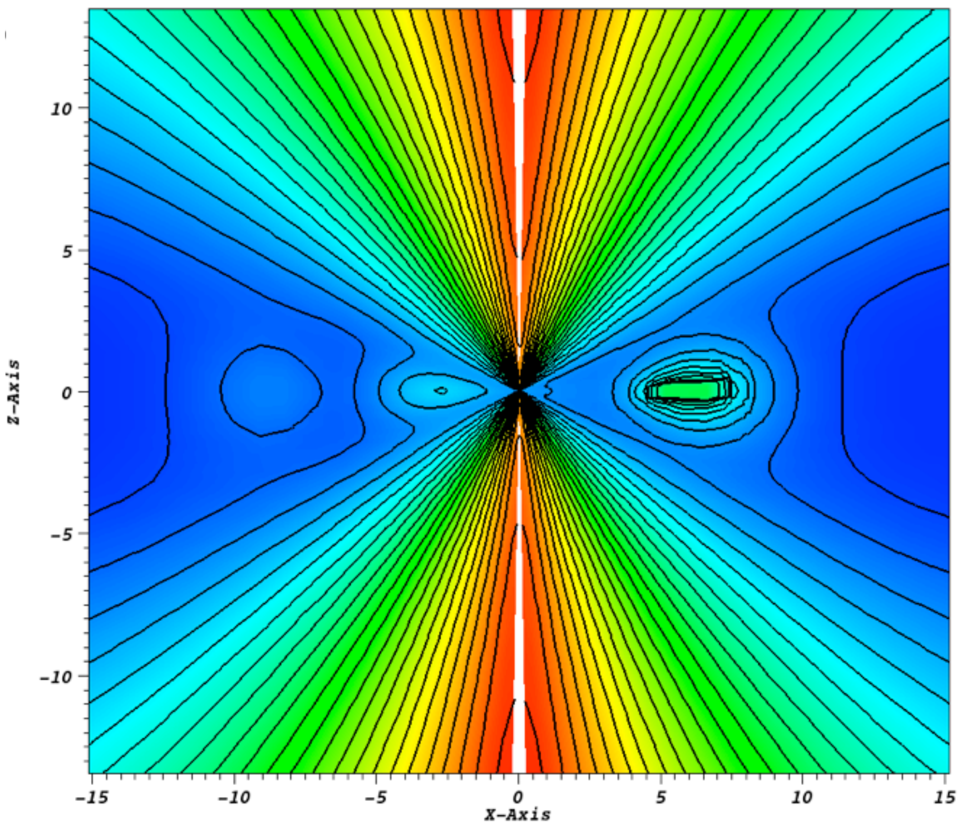

With Standard Modification:

$$F(\theta) = \quad C^{+} + C^{-} \tanh\left[(\tilde{\theta} - 90^{\circ} - \theta_F)/\Delta\theta\right]$$

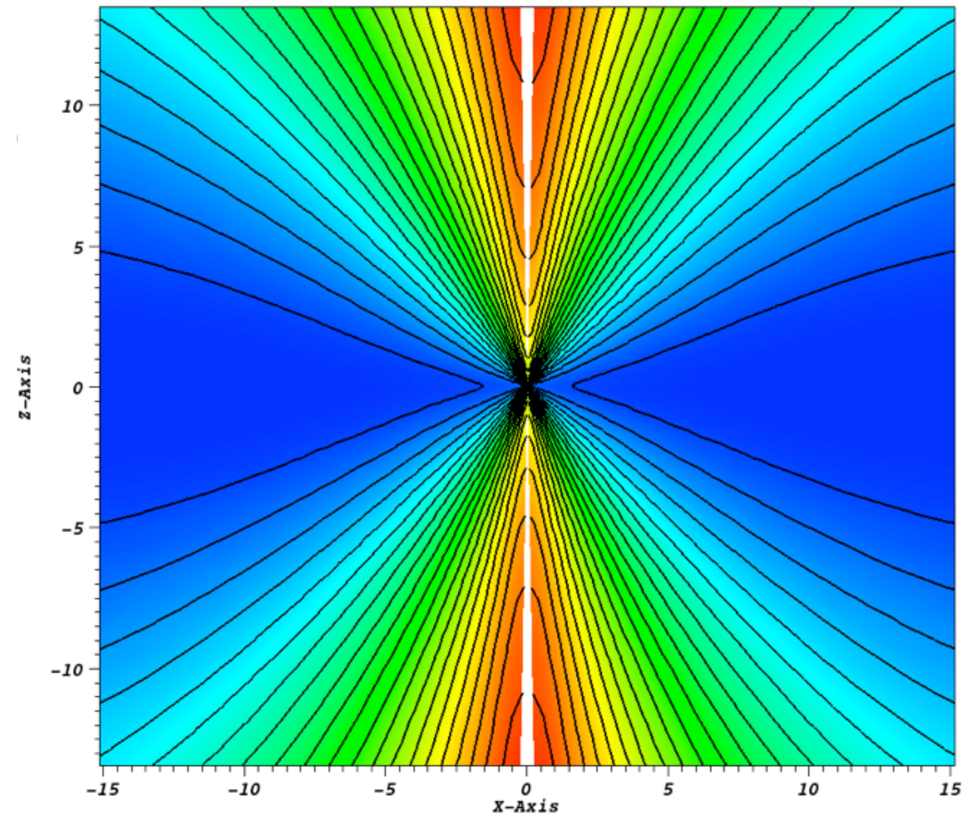



Isotropic Perpendicular Diffusion

# Results
## Jovian Electron Distribution (8 MeV)




Isotropic Perpendicular Diffusion

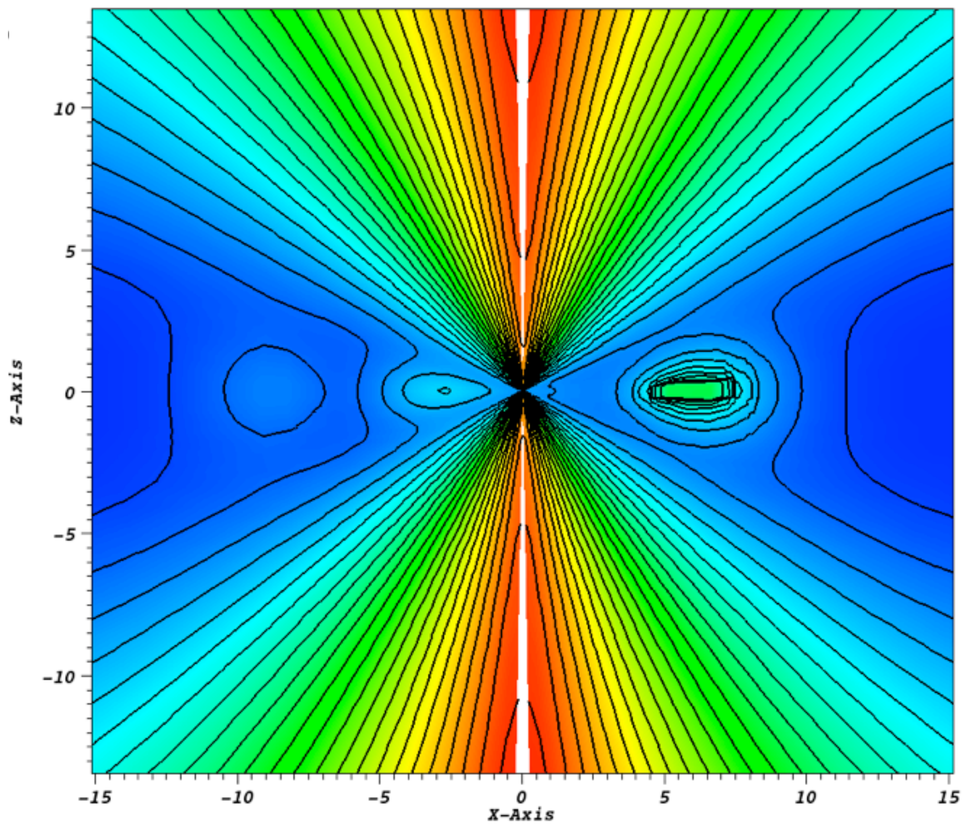



Anisotropic Perpendicular Diffusion $\kappa_{\perp 2} = 4\kappa_{\perp 1}$

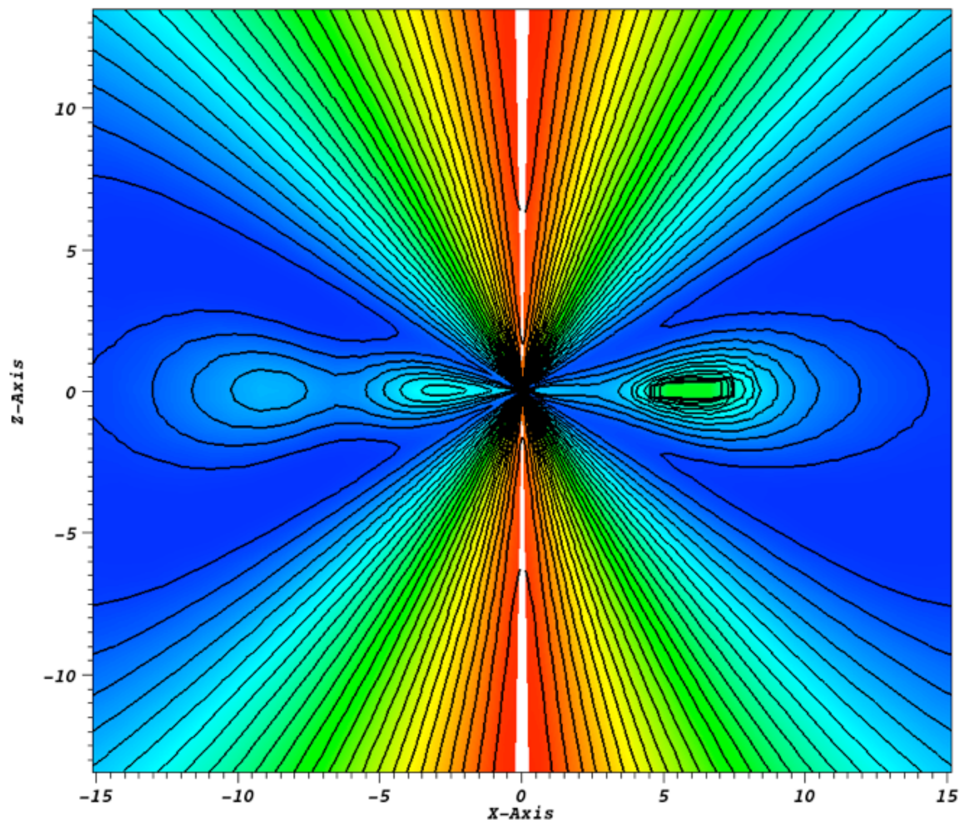
RUB

# Results

## Jovian Electron Distribution (8 MeV)



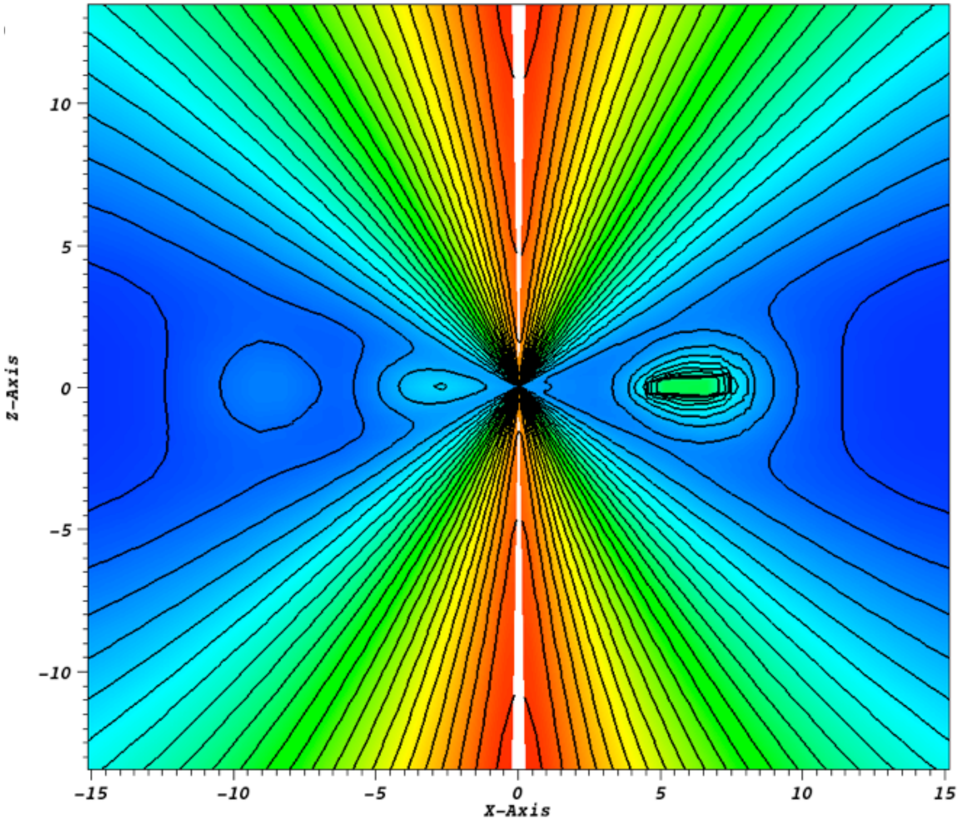

Isotropic Perpendicular Diffusion



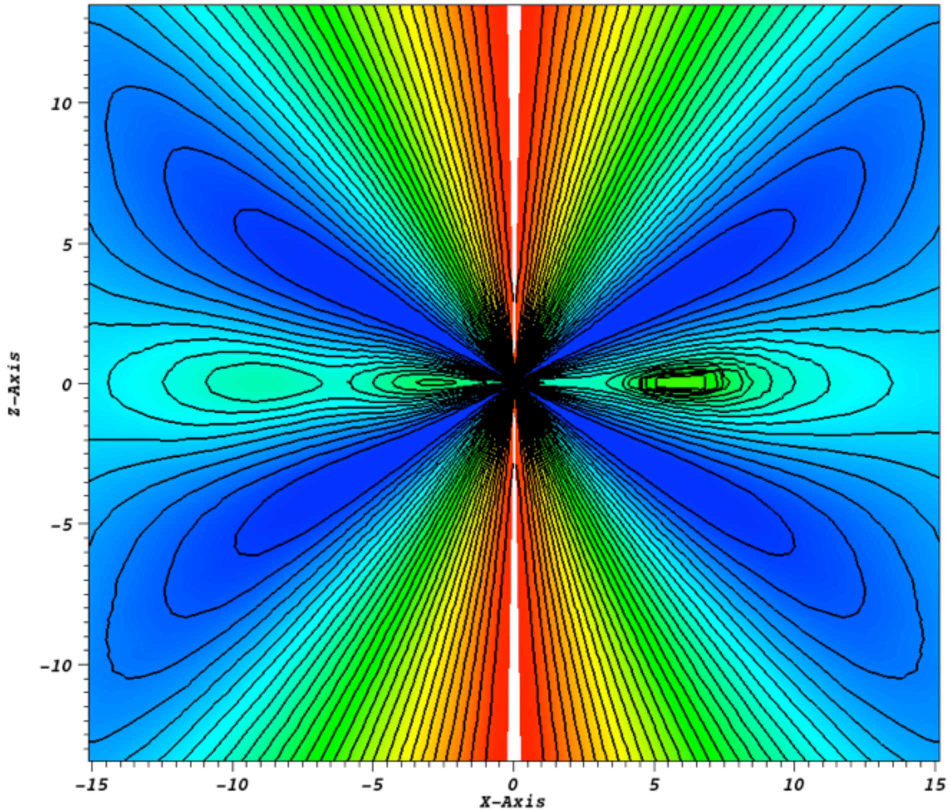

Anisotropic Perpendicular Diffusion: $\kappa_{\perp 2} = 0.5\kappa_{\perp 1}$

# Results

## Jovian Electron Distribution (8 MeV)




Isotropic Perpendicular Diffusion




Anisotropic Perpendicular Diffusion: $\kappa_{\perp 2} = 0.1 \kappa_{\perp 1}$

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# Conclusions

- Anisotropic Diffusion is important for Cosmic Ray transport on different scales and its proper treatment is needed to determine their actual distribution in the Galaxy and the modulation effects in the Heliosphere.
- Complementary numerical tools exist and are under development to investigate the properties of solutions to the Parker transport equation for various coordinate systems, setups and boundary conditions.
- Our new approach to the formulation of the diffusion tensor results in differences for the tensor elements in the fully anisotropic case and has a possible impact on the resulting modulation spectra, especially for high latitudes.
- The LIS modulation of galactic protons and the distribution of Jovian electrons shows to be sensitive to the actual structure of the diffusion tensor.