

[A new model for fully anisotropic diffusion of](http://helio_cr.tp4.rub.de/Bochum/PDF/S4-80.pdf) [CR protons and electrons in the heliosphere](http://helio_cr.tp4.rub.de/Bochum/PDF/S4-80.pdf)

Cosmic Rays and the Heliospheric Plasma Environment, Bochum, 15.09.2011

Frederic Effenberger, **Stephan Barra, Horst Fichtner, Jens Kleimann, DuToit Strauss and Klaus Scherer**

Ruhr-Universität Bochum, FAKULTÄT FÜR PHYSIK Theoretische Physik IV - Weltraum und Astrophysik

Anisotropic Diffusion of Energetic Particles in Heliospheric and Galactic Magnetic Fields

Two faces of the same coin:

Cosmic Ray modulation in the Heliosphere

Cosmic Ray transport in the Galaxy

$$
\underbrace{\frac{\partial f}{\partial t} = \nabla \cdot (\hat{\kappa} \nabla f) - \vec{v} \cdot \nabla f + p \left(\frac{1}{3} (\nabla \cdot \vec{v}) + a_{\pi} \right) \frac{\partial f}{\partial p} + 3a_{\pi} f + \frac{q}{p^2}}
$$

Anisotropic Diffusion of Energetic Particles in Heliospheric and Galactic Magnetic Fields

$$
\underbrace{\frac{\partial f}{\partial t} = \nabla \cdot (\widehat{\kappa} \nabla f) - \vec{v} \cdot \nabla f + p \left(\frac{1}{3} (\nabla \cdot \vec{v}) + \mathbf{g}'_{\pi} \right) \frac{\partial f}{\partial p} + 3 \mathbf{a}'_{\pi} f + \frac{q}{p^2}}
$$

Galactic Protons

$$
j = 12.14 \cdot \beta (E_k + 0.5E_0)^{-2.6}
$$

Now Shock

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Fig. 1. Observed and calculated intensities for hydrogen at the radial distances indicated. Spectra are offset by [Reinecke et al., 1993, JGR]

Jovian Electrons o the electrons chosen and α

tron distribution with the Jovian source absent. Be-GEOPHYSICAL RESEARCH LETTERS, VOL. 27, NO. 11, PAGES 1611-1614, JUNE 1, 2000 electrons in the hellosphere, the results of a (currently The interstellar electrons are entering the hellosphere

n the propagation of Lovian electrons in the **is limited to the sphere inside • 20-25 AU. The inner heliospheric structure. In order to heliosphere' transport modelling in 4-D phase space** On the propagation of Jovian electrons in the wore comprehensive papers.
Property probability in the papers of the papers. *Property*

Horst Fichtner

Institut für Theoretische Physik IV: Weltraum- und Astrophysik, Ruhr-Univ. Bochum, Germany

Monitor Detrictor Chofon Formains and Adri Depart **The location of the localized source is as obvious Marius Potgieter, Stefan Ferreira, and Adri Burger property** Charles and Adri Burger, and the energy in the heliocal method in the State energy in the S

Space Res. Unit, School of Physics, Potchefstroom Univ. for CHE, Potchefstroom, South Africa

$$
j_{gal}(R(p)) = \delta \exp \left(\alpha (\ln R)^2 + \beta \ln R + \gamma \right) (5)
$$

$$
j_{Jov}(E(p)) = j(E_r) \frac{E_r^{1.5} (h + E_r)^{3.5}}{E^{1.5} (h + E)^{3.5}}
$$
 (6)

 $\mathbf{P} = \mathbf{P} = \mathbf$ **Jupiter. This domain is enclosed by a heliocentric,**

- 100 -50 0 50 100 [AU] [^u]

range of the possible interstellar electron fluxes and interest interests and interests at low $\frac{1}{2}$ **energies, and for a determination of the diffusion tensor**

Two Questions, Two Complementary Numerical Tools

Heliophysical Questions:

- How are the cosmic ray protons and electrons modulated on their way from the heliospheric boundary to earth?
- Can we determine transport parameters from the propagation of Jovian electrons?
- Grid-Based Numerics: DuFort-Frankel, VLUGR3 ...

Solving the CR-Transport Equation directly on a numerical grid via finite differences or finite volume methods.

• Stochastic Differential Equations (SDE)

Solve a SDE equivalent to the Transport Equation by tracing pseudo-particles

Testing the SDE Modulation with an Analytic Parker Propagator to test the numerical codes used in this thesis agaist a (semi-) analytic solution **for a still some whose realistic case. In the solve the transport equation** with an Analyt Astron. Astrophys. 358, 347–352 (2000) ASTRONOMY

@*f*

*r*2 @*r* @*r* @*r* 3*r*² @*r* @*^p* ⁼ *S*(*r, p*) (B.1) *r*²*rr ^V* @*^f* ⁺ *^p* (*r*²*v*) **The Parker propagator for spherical solar modulation**

D. Stawicki, H. Fichtner, and R. Schlickeiser

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@

dence for the Spacial diffusion on the space of the space of the space of and momentum *p*, i.e., i.e., t. and the
Institut fur Theoretische Physik, Lehrstuhl IV: Weltraum- und Astrophysik, Ruhr-Universität Bochum, 44780 B

@

rc = Received 28 January 2000 / Accepted 30 March 2000
Received by a search wind speed of the speed of the

$$
j(r,p) = p^{2} f(r,p)
$$
\n
$$
= \frac{3}{b_{p}} \int dr_{0} \int dp_{0} \frac{S(r_{0}, p_{0})}{V(r_{0})} \frac{p_{0} y_{0}}{f_{p}} \left(\frac{r_{0}}{r}\right)^{\frac{1+\beta_{p}}{2}}
$$
\n
$$
\times \left(\frac{p_{0}}{p}\right)^{\frac{3\beta_{p}-4\alpha_{p}-5}{2(2+\alpha_{p})}} \exp\left(-\frac{y_{0}(1+h_{p}^{2})}{f_{p}}\right)
$$
\n
$$
\times I_{\frac{1+\beta_{p}}{1+\alpha_{p}-\beta_{p}}} \left(\frac{2y_{0}h_{p}}{f_{p}}\right)
$$
\n
$$
\times I_{\frac{1+\beta_{p}}{1+\alpha_{p}-\beta_{p}}} \left(\frac{2y_{0}h_{p}}{f_{p}}\right)
$$

17 have to be included in all cases to reproduce observations (see, ACRs and GCRs, respectively. **Fig. 1.** The modulated spectra of ACRs and GCRs in the heliosphere. The solar wind termination shock marking the position of the solites
is located at $r_{\rm sh} = 100 \,\rm AU$. The solid lines are the combined spectra, is focated at $r_{\rm sh} = 100 \text{ AU}$. The solid lines are the combined spectration the dotted and dashed lines indicate the individual contributions from e.g., Reinecke et al. 1993; le Roux & Fichtner 1993; le Roux & Fichtner 1997). In the Roux Andrew 1997 The solar wind termination shock marking the position of the sources

Already Parker (1965) has given a variety of solutions of

Many studies have been carried out to derive the interstellar proton spectrum (for a recent compilation see, e.g., Fig. 6 in \mathcal{L} 1997). We selected the one obtained by Webber et al.

Results Testing the SDE Modulation with an Analytic Parker Propagator

v · ⇤*f* + *p*

$$
\begin{pmatrix}\n\frac{\partial f}{\partial t} = \nabla \cdot (\hat{\kappa} \nabla f) \\
\text{local Tensor} \\
\hat{\kappa}_L = \begin{pmatrix}\n\kappa_{\perp 1} & 0 & 0 \\
0 & \kappa_{\perp 2} & 0 \\
0 & 0 & \kappa_{\parallel}\n\end{pmatrix}
$$

Anisotropic Diffusion The "Classical" Approach: Euler-Angle Transformation $\mathbf{F} = \mathbf{F} \cdot \mathbf{F} = \mathbf{F} \cdot \mathbf{F} \cdot \mathbf{F} = \mathbf{F} \cdot \mathbf{F} \cdot \mathbf{F} = \mathbf{F} \cdot \mathbf{F} \cdot \mathbf{F} \cdot \mathbf{F} = \mathbf{F} \cdot \$ λ nisotropic Diffusion from field-aligned to spherical coordinates, resulting in the elements given λ

$$
\kappa_{rr} = \kappa_{\perp,2} \sin^2 \zeta + \cos^2 \zeta \left(\kappa_{||} \cos^2 \Psi + \kappa_{\perp,3} \sin^2 \Psi\right),
$$

\n
$$
\kappa_{r\theta} = -\kappa_A \sin \Psi + \sin \zeta \cos \zeta
$$

\n
$$
\times \left(\kappa_{||} \cos^2 \Psi + \kappa_{\perp,3} \sin^2 \Psi - \kappa_{\perp,2}\right),
$$

\n
$$
\kappa_{r\phi} = -\kappa_A \cos \Psi \sin \zeta - \left(\kappa_{||} - \kappa_{\perp,3}\right) \sin \Psi \cos \Psi \cos \zeta,
$$

\n
$$
\kappa_{\theta r} = \kappa_A \sin \Psi + \sin \zeta \cos \zeta
$$

\n
$$
\times \left(\kappa_{||} \cos^2 \Psi + \kappa_{\perp,3} \sin^2 \Psi - \kappa_{\perp,2}\right),
$$

\n
$$
\kappa_{\theta\theta} = \kappa_{\perp,2} \cos^2 \zeta + \sin^2 \zeta \left(\kappa_{||} \cos^2 \Psi + \kappa_{\perp,3} \sin^2 \Psi\right),
$$

\n
$$
\kappa_{\theta\phi} = \kappa_A \cos \Psi \cos \zeta - \left(\kappa_{||} - \kappa_{\perp,3}\right) \sin \Psi \cos \Psi \sin \zeta,
$$

\n
$$
\kappa_{\phi r} = \kappa_A \cos \Psi \sin \zeta - \left(\kappa_{||} - \kappa_{\perp,3}\right) \sin \Psi \cos \Psi \cos \zeta,
$$

\n
$$
\kappa_{\phi\theta} = -\kappa_A \cos \Psi \cos \zeta - \left(\kappa_{||} - \kappa_{\perp,3}\right) \sin \Psi \cos \Psi \sin \zeta,
$$

\n
$$
\kappa_{\phi\phi} = \kappa_{||} \sin^2 \Psi + \kappa_{\perp,3} \cos^2 \Psi,
$$

\n(17)
\nwhere $\tan \Psi = -B_{\phi}/\left(B_r^2 + B_\theta^2\right)^{1/2}$ and $\tan \zeta = B_\theta/B_r$. Note that

Alania & Dzhapiashvili (1979), Alania (2002), and Kobylinski

Serret-Frenet relations (e.g., [Marris and Passman, 1969]):

THE SALE OF STATE OF LOCAL FIELD DIRECTION Anisotropic Diffusion The new Approach: Local Frenet-Trihedron

*B*²

 T ocalculate a second unit vector from the local field-aligned trihedron, one needs the local field-aligned tri

$$
\left(\text{Tangential Vector }\ \vec{t} = \frac{\vec{B}}{B}\right)
$$

Serret-Frenet relations (e.g., [Marris and Passman, 1969]):

$$
(\vec{t} \cdot \nabla)\vec{t} = k\vec{n} \; ; \; (\vec{t} \cdot \nabla)\vec{n} = -k\vec{t} + \tau\vec{b} \; ; \; (\vec{t} \cdot \nabla)\vec{b} = -\tau\vec{n}
$$

To calculate a second unit vector from the local field-aligned trial field-aligne

as well as

Binormal Vector $\vec{b} = \vec{t} \times \vec{n}$

$$
\begin{pmatrix}\n\times \vec{n} \\
A = \begin{pmatrix}\nn_1 & b_1 & t_1 \\
n_2 & b_2 & t_2 \\
n_3 & b_3 & t_3\n\end{pmatrix}\n\end{pmatrix}
$$

The global tensor ˆ*^G* can be calculated by the usual tensor transformation rule (see

Anisotropic Diffusion The General Transformation rule (see The Usual tensor transformation rule (see The Usual Transformation rule (see Th

The symmetric global difusion tensor: ˆ*^G* = *A*ˆ*LA*¹ = *A*ˆ*LA^T* (3.3) The symmetric global dinasion tensor.

$$
\hat{\kappa}_{11} = \kappa_{\perp 1} n_1^2 + \kappa_{\perp 2} b_1^2 + \kappa_{\parallel} t_1^2
$$
\n
$$
\hat{\kappa}_{12} = \kappa_{\perp 1} n_1 n_2 + \kappa_{\perp 2} b_1 b_2 + \kappa_{\parallel} t_1 t_2
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\n
$$
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$$

General magnetic field & local diffusion coefficients

The symmetric global difusion tensor: ˆ*^G* = *A*ˆ*LA*¹ = *A*ˆ*LA^T* (3.3) The symmetric global diffusion tensor.

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$$

General magnetic field & local diffusion coefficients Calculate local trihedron Include in transformation to global reference frame General, fully anisotropic, global diffusion tensor

RUB

The symmetric global difusion tensor: ˆ*^G* = *A*ˆ*LA*¹ = *A*ˆ*LA^T* (3.3) The symmetric global diffusion tensor.

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\n
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$$

In any case: The simple structure of the difusion tensor in the local frame may lead to complicated tensor elements in the global frame, depending on the magnetic field.

fieldline moving on the cone, because ↵*e* is always perpedicular. \mathbb{R}^n and \mathbb{R}^n interplanetary magnetic field is the interpreta

the green line of the binomal vector field. The traditional vector field. The traditional \mathbf{r}_i normal and binormal vector fields. One can see the "traditional" normal vector

Anisotropic Diffusion The Parker-Field Tensor Elements (isotropic perp.)

$$
\begin{cases} \kappa_{\parallel}=1,\,\kappa_{\perp 1}=0.2,\,\kappa_{\perp 2}=0.2 \\ r=10 \mathrm{AU},\,\varphi=\pi \end{cases}
$$

Anisotropic Diffusion

The Parker-Field Tensor Elements (anisotropic perp.)

$$
\begin{cases} \kappa_{\parallel} = 1, \, \kappa_{\perp 1} = 0.05, \, \kappa_{\perp 2} = 0.2 \\ r = 10 \text{AU}, \, \varphi = \pi \end{cases}
$$

Results Modulation Spectra Isotropic vs Anisotropic

Results Modulation Spectra Isotropic vs Anisotropic (Normalized to LIS)

Results Effect of Different Tensor Formulations

Isotropic perpendicular diffusion 1 AU Earth spectra

Anisotropic perpendicular diffusion 1 AU Earth spectra

80

60

 $40 -$

20

 \mathbf{o}

 -20

 -40

 -60

 -100

 $Z - A x i s$

Results Jovian Electron Distribution (8 MeV)

With Standard Modification:

1

 -50

$$
F(\theta) = \left[C^+ + C^- \tanh \left[(\tilde \theta - 90^\circ - \theta_F)/\Delta \theta \right] \right]
$$

Isotropic Perpendicular Diffusion

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With Standard Modification:

$$
F(\theta) = \hspace{0.2cm} C^+ + C^- \tanh \left[(\tilde \theta - 90^\circ - \theta_F)/\Delta \theta \right]
$$

Isotropic Perpendicular Diffusion

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RUB

Conclusions

- Anisotropic Diffusion is important for Cosmic Ray transport on different scales and its proper treatment is needed to determine their actual distribution in the Galaxy and the modulation effects in the Heliosphere.
- Complementary numerical tools exist and are under development to investigate the properties of solutions to the Parker transport equation for various coordinate systems, setups and boundary conditions.
- Our new approach to the formulation of the diffusion tensor results in differences for the tensor elements in the fully anisotropic case and has a possible impact on the resulting modulation spectra, especially for high latitudes.
- The LIS modulation of galactic protons and the distribution of Jovian electrons shows to be sensitive to the actual structure of the diffusion tensor.