A stochastic approach to galactic propagation

I. Büsching[1], **A. Kopp**[2], F. Effenberger[1], R. D. Strauss[3],
M. S. Potgieter[3], H. Fichtner[1]

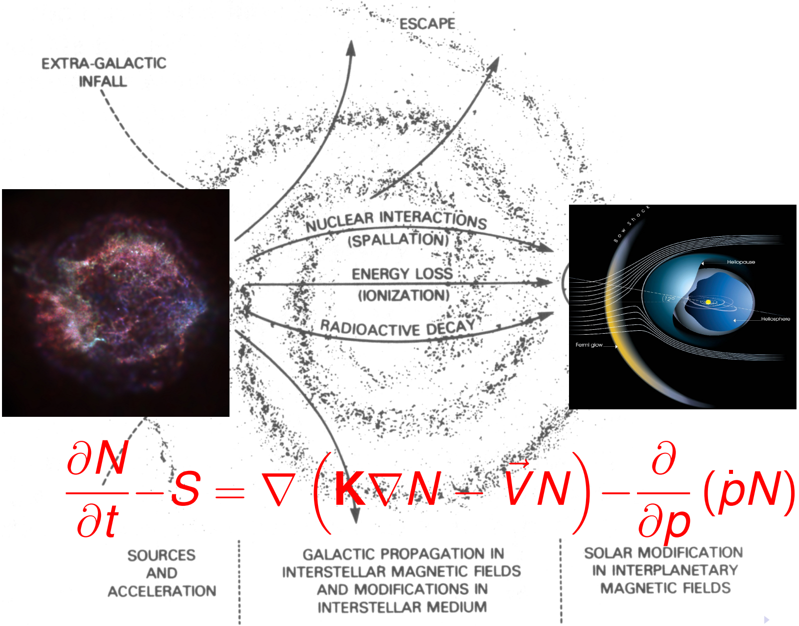
[1]Theoretische Physik IV, Ruhr-Universität Bochum

[2]IEAP, Christian-Albrechts-Universität zu Kiel, Germany

[3]Centre for Space Research, North-West University, Potchefstroom, South Africa

Bochum, September 12th 2011

Galactic Propagation of Cosmic Rays (CRs)



$$\frac{\partial N}{\partial t} - S = \nabla\left(\mathbf{K}\nabla N - \vec{V}N\right) - \frac{\partial}{\partial p}\left(\dot{p}N\right) - \frac{N}{T}$$

Problem:

Supernovae are point-like, transient sources

- 3D time dependent propagation model needed
- Numerical expensive

- Good knowledge of local sources is needed

Stochastic Differential Equations (SDE)

Idea: solve transport equation (TPE) by propagating a large ensemble of pseudo particles and bin the results to obtain the distribution function N

Time-forward propagation: write TPE in conservative form:

$$\frac{\partial N}{\partial t} = -\sum_i \frac{\partial}{\partial x_i} \left(A_i(\mathbf{x}, t)N\right) + \frac{1}{2}\sum_{i,j} \frac{\partial^2}{\partial x_i \partial x_j} \left(C_{i,j}(\mathbf{x}, t)N\right)$$

- plus: source (S) and loss (L) terms
- available also in spherical and cylindrical coordinates
- general symmetric diffusion tensor $\underline{C}$ with all non-diagonal elements being allowed to be $\neq 0$
- sum includes the momentum p as a fourth dimension

Stochastic Differential Equations (SDE)

Time-backward propag.: write TPE in non-conservative form:

$$\frac{\partial N}{\partial t} = -\sum_i A_i(\mathbf{x}, t)\frac{\partial N}{\partial x_i} + \frac{1}{2}\sum_{i,j} C_{i,j}(\mathbf{x}, t)\frac{\partial^2 N}{\partial x_i \partial x_j}$$

Both forms of the TPE are equivalent to the set of SDEs:

$$d\mathbf{x} = \mathbf{A}\left(\mathbf{x}, t\right) dt + \underline{\mathbf{B}}\left(\mathbf{x}, t\right) \cdot d\mathbf{W}(t)$$

with:

$$\underline{\mathbf{C}} = \underline{\mathbf{B}} \cdot \underline{\mathbf{B}}^{\mathbf{T}}$$

$$d\mathbf{W}(t) = \sqrt{dt}\,\mathbf{n}(t),$$

where $\mathbf{n}(t)$ is a vector of normally distributed random numbers.

Stochastic Differential Equations (SDE)

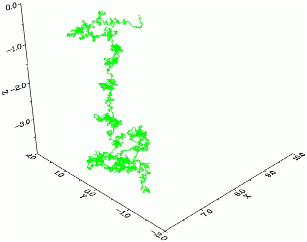
- Very stable
- No numerical grid needed
- Choice of time step Δt determines spatial resolution
- Embarrassingly parallel problem
- Solvable forward/backward in time, depending on problem
- Literature:

  Gardiner, Handbook of Stochastic Methods; Øksendal, Stochastic Differential Equations; Kloeden & Platen, Numerical Solution of Stochastic Differential Equations

SDE for 3D Galactic propagation

SDE code so far solves the equation:

$$\frac{\partial N}{\partial t} - S = \nabla \cdot (\underline{\mathbf{K}} \cdot \nabla N - \mathbf{V} N) - \frac{\partial}{\partial p}(\dot{p} N) - L\, N$$



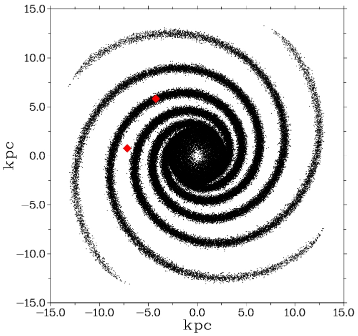
- all elements of $\underline{\mathbf{K}}$, $\mathbf{V}$, $\dot{p}$, S, and L depend on $\mathbf{r}$, p and t (4+1 D), i.e. code capable of calculation in 1,2 or 3 spatial dimensions, momentum (or energy) and time
- implemented in C, version for scalar diffusion adapted to run on GPU with CUDA

S is a real particle source or a boundary condition

L is taken into account by a “path amplitude” (weighting)

Spectral variation inside and outside of spiral arms

cosmic ray proton flux inside and outside spiral arms:



- Diffusion coefficient:

$$k = \begin{cases} k_0 \left(\frac{\zeta}{\zeta_0}\right)^{0.6} & \text{for } \zeta > \zeta_0 \\ k_0 \left(\frac{\zeta}{\zeta_0}\right)^{-0.48} & \text{for } \zeta < \zeta_0 \end{cases}$$

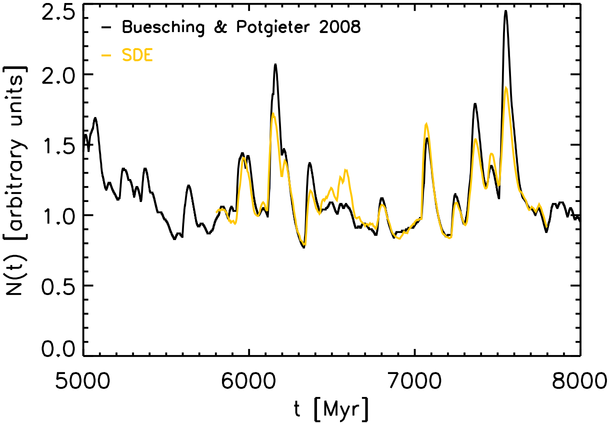
$$k_0 = 0.027\,\mathrm{kpc}^2\mathrm{Myr}^{-1}$$
$$\zeta_0 = 4\,GV/c$$

- $H = 4\,\mathrm{kpc}$

- 130001 transient point sources clustering in spiral arms

(see Büsching & Potgieter 2008)

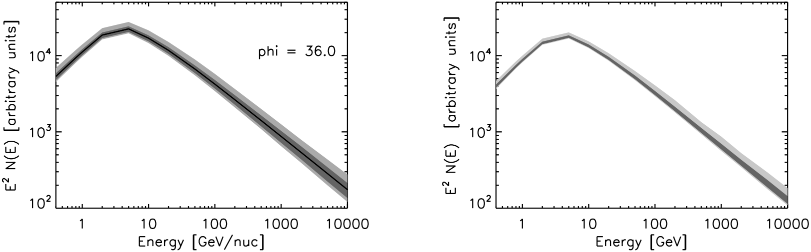
Comparison with Büsching & Potgieter 2008



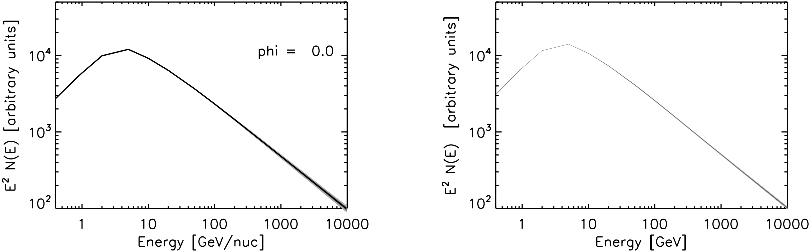
Temporal variation of the cosmic ray proton flux at 10 GeV (inside spiral arm).

Spectral variation inside and outside of spiral arms

CR proton spectra inside spiral arms:



CR proton spectra outside spiral arms:



Büsching & Potgieter 2008 SDE code

Spatial variable diffusion coefficient

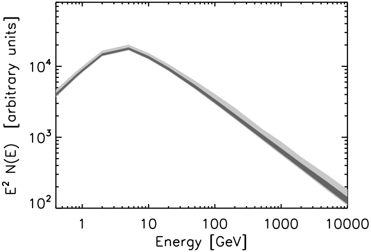
- Level of turbulence higher inside spiral arms (inarm region) than in interarm region
- Particles diffuse slower inside spiral arms
- Assume diffusion coefficient to be smaller inside spiral arms

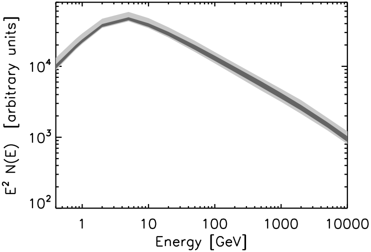
$$k_{sv} = k_{sc}\left(1 - 0.6\exp\left[-\left(\frac{d}{0.3\,kpc}\right)^2\right]\right),$$

with d the distance from nearest spiral arm.

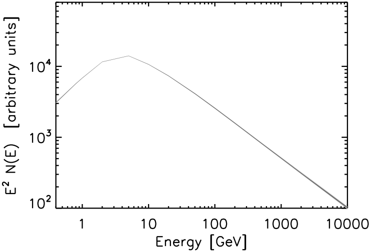
Spectral variation inside and outside of spiral arms for spatial variable diffusion coefficient

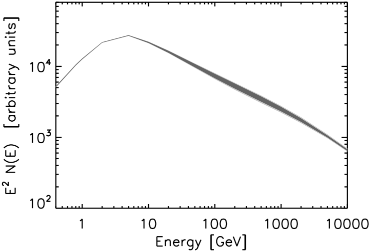
CR proton spectra inside spiral arms:





CR proton spectra outside spiral arms:





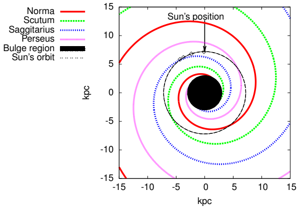
spatially constant diffusion coefficient

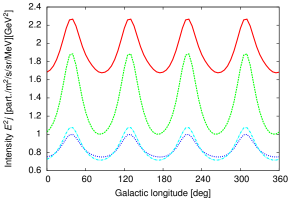
spatially variable diffusion coefficient

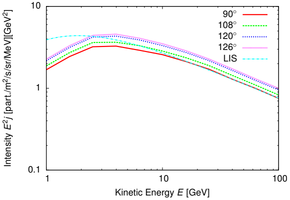
Comparison isotropic/anisotropic case

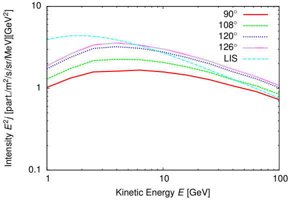
Local diffusion tensor ($\kappa_{\perp 1,2} = 0.1k$, $\kappa_{\|} = k$):

$$\underline{K} = \begin{pmatrix} \kappa_{\perp 1} & 0 & 0 \\ 0 & \kappa_{\perp 2} & 0 \\ 0 & 0 & \kappa_{\|} \end{pmatrix}, \text{ where } \mathbf{B} = \frac{1}{r}(\sin\psi\, \mathbf{e}_r + \cos\psi\, \mathbf{e}_\varphi)\, e^{-\left(\frac{z}{\sigma_z}\right)^2}$$









isotropic case (—, — ↗) anisotropic case(—, — ↑)

Summary

- Full 3D time-dependent propagation models have to be used for primary cosmic rays, if they originate in transient, point-like sources i.e. supernovae.
- Stochastic Differential Equations (SDEs) provide an robust tool to integrate Fokker-Planck type equations.
- We developed an SDE code that
  - solves the Cosmic Ray TPE in up to 3 spatial dimensions, momentum and time
  - compares well with previous 3D capable codes
  - can tackle anisotropic diffusion tensors
  - can take advantage of the computational power of modern graphics processing units (GPU) using CUDA.