# Change from sub- to superdiffusion in charge-fluctuating dusty plasmas

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# Anomalous diffusion

"normal" diffusion equation for distribution function $f(x,t)$:

$$\begin{aligned} \frac{\partial f}{\partial t} &= D\frac{\partial^2 f}{\partial x^2} \\ \rightarrow \Delta x &\propto \sqrt{\Delta t} \end{aligned}$$

generalisation: anomalous diffusion with fractional derivatives:

$$\begin{aligned} \frac{\partial^\alpha f}{\partial t^\alpha} &= F\frac{\partial^\beta f}{\partial x^\beta} \\ \rightarrow \Delta x &\propto \sqrt{\Delta t}^{\,m} \end{aligned}$$

($D$: diffusion coefficient, $F$: fractional diffusion coefficient)

with the exponent $m = \frac{2\alpha}{\beta}$.

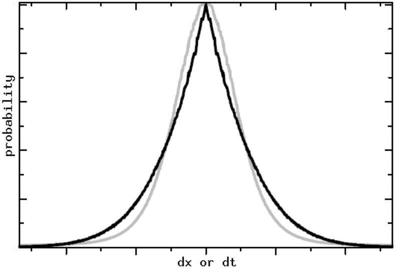
- $m < 1$ : subdiffusion
- $m = 1$ : quasidiffusion (normal diffusion only if $\alpha = 1$ and $\beta = 2$)
- $m > 1$ : superdiffusion

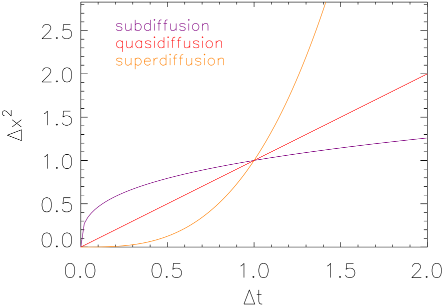
# Anomalous diffusion

$m < 1$ subdiffusion:
distribution of **waiting times** has a wide tail
$\rightarrow$ enhanced probability for large $\Delta t$
$\rightarrow$ intuitively related to small (or zero) $\Delta x$

$m = 1$ normal diffusion: $\rightarrow$ Gaussian distribution

$m > 1$ superdiffusion:
distribution of **walking distances** has a wide tail (Lévy flights)
$\rightarrow$ enhanced probability for large $\Delta x$



# Charge-fluctuating dust in the ISM

Example:

- plasma in the interstellar medium (ISM): electrons, ions and neutral particles with a small contribution of dust grains
- dust grains may be regarded as test particles
- dust charge $Z$ determined self-consistently as a function of densities, temperatures and external UV radiation: $Z = Z(n_{\mathrm{e,i}}, T_{\mathrm{e,i}}, \Phi)$
- for temperatures $T_e \sim 0.01$ eV we obtain a time-average $|\langle Z \rangle| \sim 1$ or even less

$\rightarrow$ $Z$ may fluctuate, e.g. between $Z = 0$ and $Z = -1$

general case: $Z$ can assume values $Z = 0$, $Z = \pm 1$, $Z = \pm 2$, ...

# The numerical experiment

- Question: how does a fluctuating charge of the dust grains influence their motion (perpendicular diffusion)?
- 2D configuration $(x, y)$ with magnetic field $B_z \perp$ to the plane
- We start with $Z = -1$ and randomly change the charge by $Z \to Z \pm \Delta Z$ (currently $\Delta Z = 1$)
- $Z$ is allowed to assume values $Z_{\min} \leq Z \leq Z_{\max}$
- Control parameter:

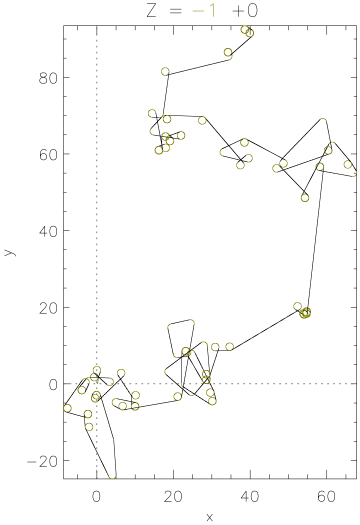
$$q = \frac{\tau_{\mathrm{gyr}}}{\tau_{\mathrm{chg}}}$$

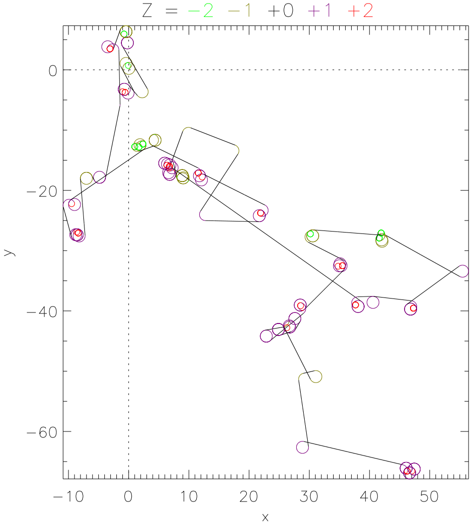
  - with (initial) gyration time $\tau_{\mathrm{gyr}} = \frac{|Z| e B_z}{2\pi m}$
  - and time scale of charge fluctuations, $\tau_{\mathrm{chg}}$

$\longrightarrow$ $q$ gives the average number of recharging events per gyration

# The numerical experiment

example trajectories for $q = 1$ ($\cdots$: initial position):




→ competition between gyration and free flights

# Validation of the code

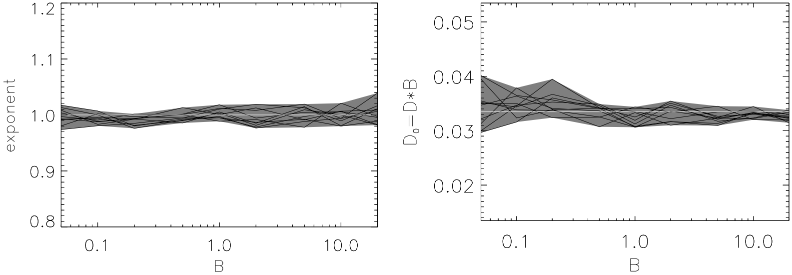
- the numerical code analytically integrates the equation of motion:

$$m\frac{d\vec{v}}{dt} = Ze\vec{v} \times \vec{B}$$

  - gyration for $Z \neq 0$
  - free flight for $Z = 0$ ($\vec{v}$ =const.)
- we study the distance of the guiding center as a function of time (the guiding center is propagated parallel to the particle if $Z = 0$)
- 10 simulation runs with an ensemble of 1001 (test) particles each

# Validation of the code

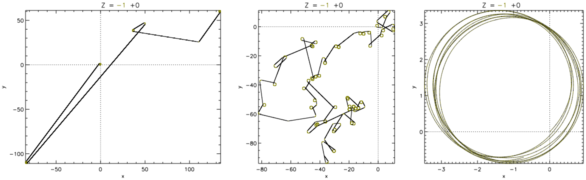
- test case: $Z = -1$ is kept fixed, instead: velocity perturbations
- determination of parameters $m$ (exponent) and $D$ (diffusion coefficient) as a function of $B_z$:



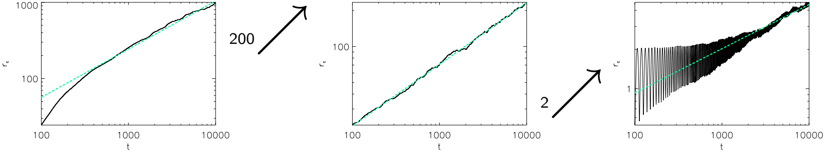

- we obtain normal (Bohm) diffusion with $D \cdot B_z \approx$ const., i.e. $D \propto \frac{1}{B_z}$

# Results

- diffusive behaviour as a function of $q$ for fixed $B_z$ and $Z = -1$ or 0: three cases: $q = 0.05$, $q = 1$, $q = 500$:
  - sample trajectories:



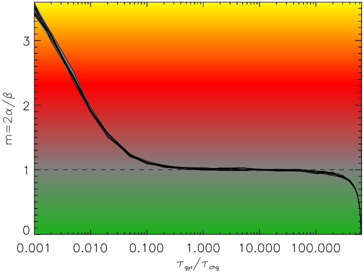

  - fit of exponents $m$ (note the different scales of the ordinate):




$m = 1.266$ (superdiff.), $m = 1.029$ (quasidiff.), $m = 0.679$ (subdiff.)

# Results

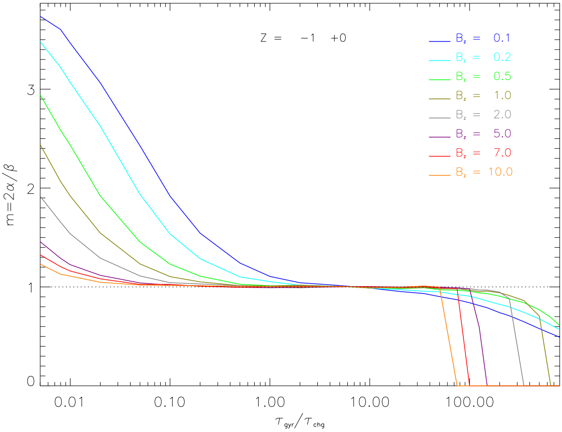
- exponent $m$ as a function of $q$:




- **transition from subdiffusion via a wide plateau of quasidiffusion to superdiffusion by changing the single parameter $q$!**

# Outlook

- dependence on the magnetic field:

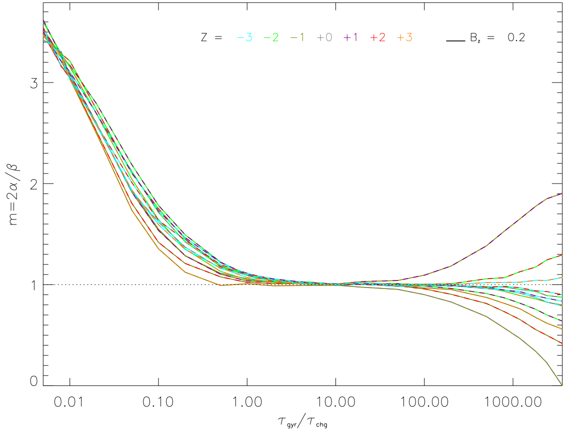



- larger effect for smaller $B_z$
- "cut-off" at high $q$ due to limited time-step

  (charge changes at each time step → (larger) gyration motion)

# Outlook

- dependence on the allowed charge states:




- strongest subdiffusion at high $q$ for $Z = -1/0$
- superdiffusion at high $q$ if $Z_{\mathrm{min}} = -Z_{\mathrm{max}}$