MHD Turbulence in the Solar Wind

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Outline

A. Solar Wind Fluctuations
   1. Nature. waves? turbulence? …
   2. Anisotropy
   3. Models: ~ critical balance

B. Radial evolution
   - Turbulence models
   - Observational comparison

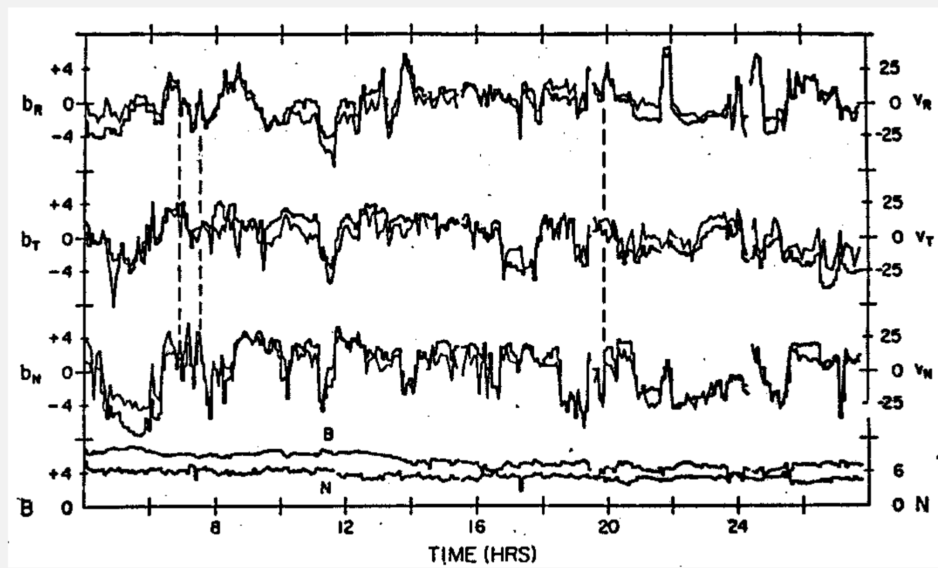
incompressible
Hc = <v.b> = 0

[redacted]

- Observations suggest presence of waves and turbulence

Alfven Waves

[Belcher & Davis 1971]

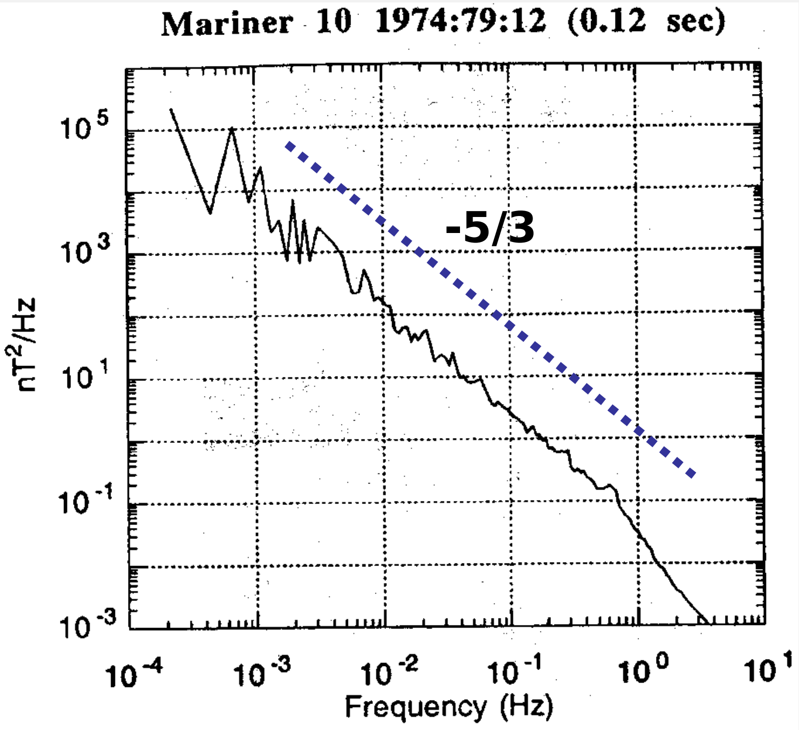


i.e., v = b which **“implies”** Alfven waves

Turbulence:

Magnetic Energy Spectrum, P(f)

Power-law inertial range



[Goldstein EA 1995]

Aside: Kolmogorov Theory

statistically steady

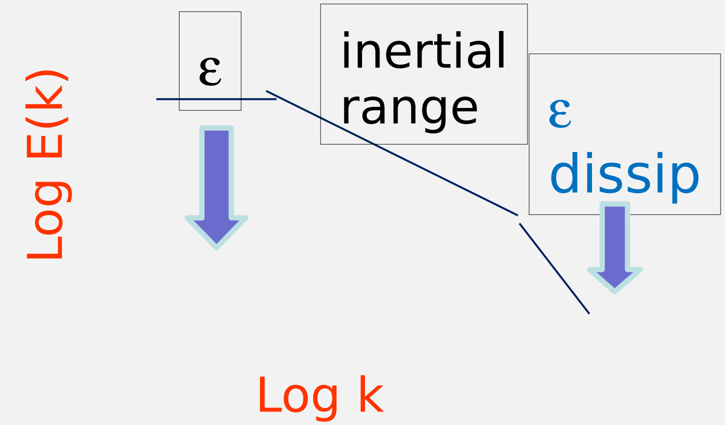
nonlinear terms conserve E

energy flux local in *k*-space, so

$$\varepsilon \quad \propto \quad k^a E(k)^b$$

Dimensional analysis gives inertial range form:

$$E(k) = C_K \varepsilon^{2/3} k^{-5/3}$$



- Write fields as mean + fluct

$$B = \langle B \rangle + b$$

- Often

  b / <B> ~ ½

  so <B> is significant, but NOT dominant

- Anisotropy

Variance Anisotropy

- <B>-aligned coords
- perp power dominates:

bx2 : by2 : bz2 = 5 : 4 : 1

Vsw

θ <B>

  – BelcherDavis71, KleinEA91, HorburyEA95,…

- Interpretations?

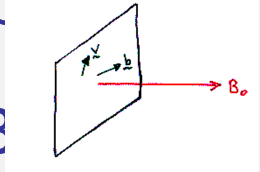
A: `slab’ ||-prop. Alfven waves

ampl ? <B>

k || <B>

B: quasi-2D turb

ampl, k ? <B



–No conclusion from min variance dirn

Spectral/Correlation Anisotropy

Correlation Anisotropy: Maltese cross

- MatthaeusEA90:
  - ***b*** flucts at 1AU
  - Construct corrn fn

    Rbb(r**k**, r**?**) wrt <B>

Find
- No single symmetry.
- Suggests 2

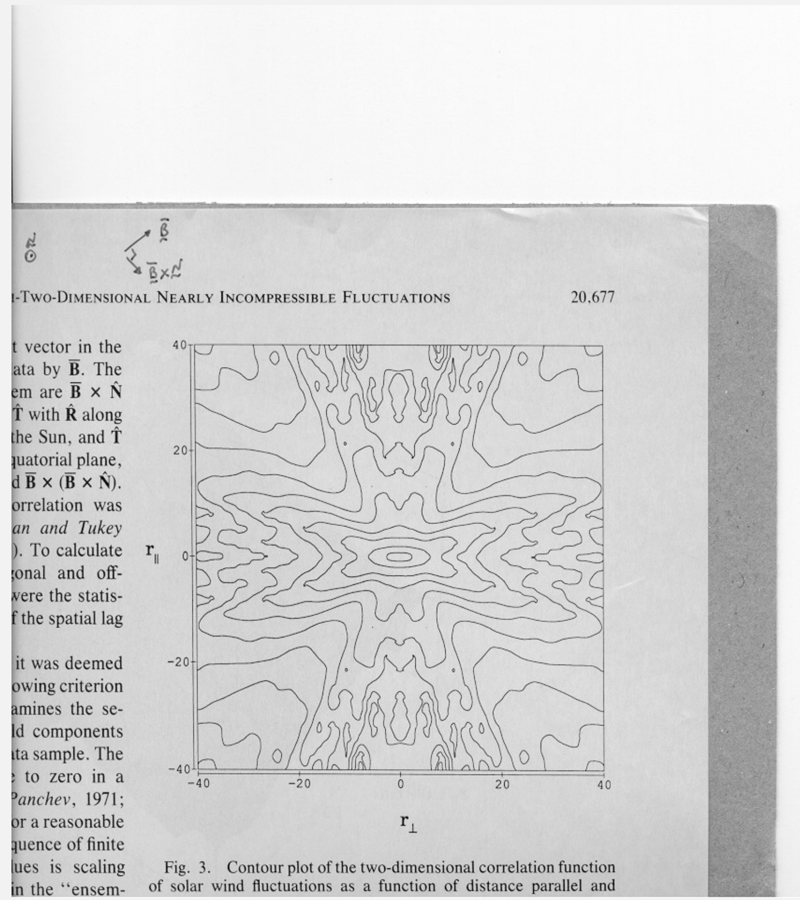
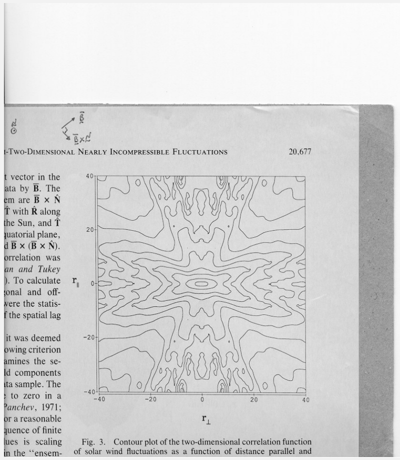


Fig. 3. Contour plot of the two-dimensional correlation function of solar wind fluctuations as a function of distance parallel and

- DassoEA05 update
  - At 1AU, slow and fast wind give different Rbb corrn fns


-Two-Dimensional Nearly Incompressible Fluctuations 20,677

t vector in the
ata by $\bar{\mathbf{B}}$. The
em are $\bar{\mathbf{B}} \times \hat{\mathbf{N}}$
$\hat{\mathbf{T}}$ with $\hat{\mathbf{R}}$ along
the Sun, and $\hat{\mathbf{T}}$
quatorial plane,
d $\bar{\mathbf{B}} \times (\bar{\mathbf{B}} \times \hat{\mathbf{N}})$.
orrelation was
an and Tukey
). To calculate
gonal and off-
were the statis-
f the spatial lag

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Panchev, 1971;
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Fig. 3. Contour plot of the two-dimensional correlation function of solar wind fluctuations as a function of distance parallel and

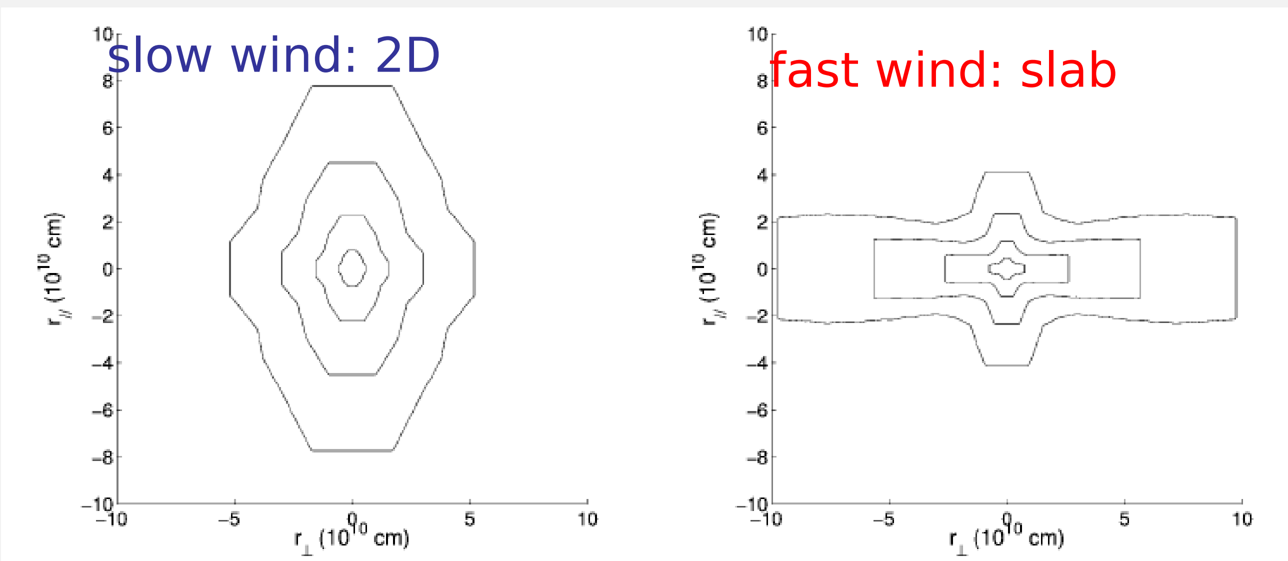


FIG. 1.—Level contours for $R_{bb}(r)$. *Left*, slow solar wind ($V_{sw} < 400$ km s^{-1}); *right*, fast solar wind ($V_{sw} > 500$ km s^{-1}). (See text.) Levels are at 1200, 1400, 1600, and 1800 km^2 s^{-2}.

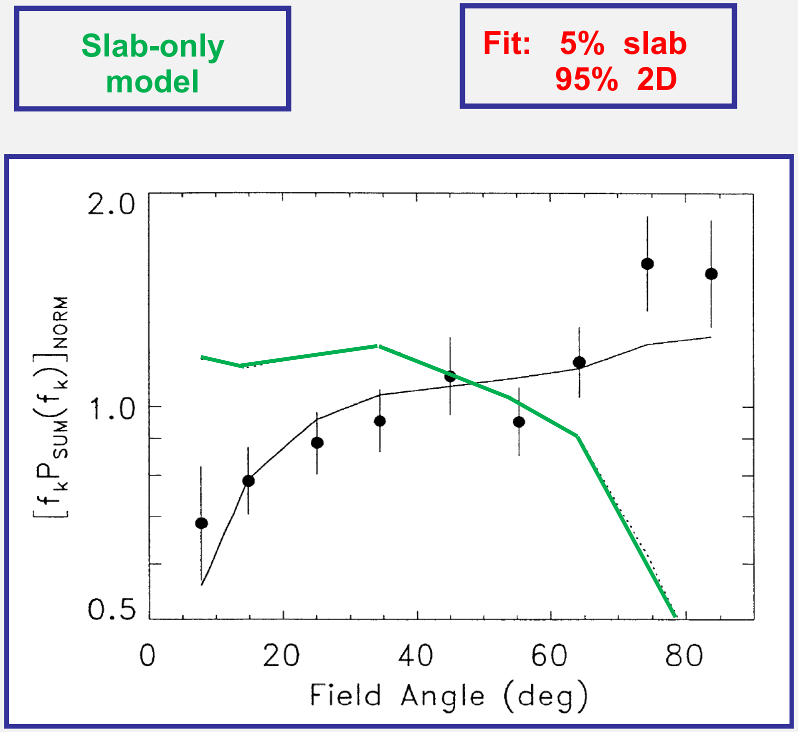
Interp: Slow/older wind has had more time to evolve [to

Spectral anisotropy

- Observe magnetic energy spectra, P(f)
- At different angles to <***B*** >

- Assume power-law inertial ranges
- Fit to model: *slab + 2D*

- Best fit:
  80% 2D, 20% slab



Bieber et al., J. Geophys. Res., 1996

Weak Turb explanation for anisotropy

- <B> causes suppression of || transfer
- leads to lpar > lperp
  - plasma devices ~10x
  - SW measurements ~ 3x

    (WeygandEA09)

Anisotropy when B0 ≠ 0 [ShebalinEA83, GaltierEA00]

Weak turb approach:

- 0th order: Alfven waves $\boldsymbol{v}, \boldsymbol{b} \sim e^{i(\boldsymbol{k}\cdot\boldsymbol{x} \pm \omega t)}$
- 1st order: nonlinear corrections
  - $\dot{\boldsymbol{v}}_k \sim \boldsymbol{v}_q \cdot \nabla \boldsymbol{v}_p$, etc

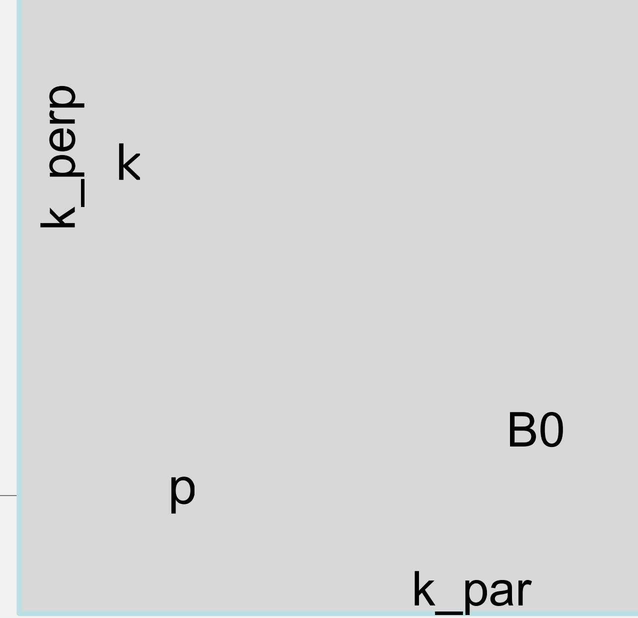
  require

  - ***k*** = ***p*** + ***q***
  - and k|| = p|| - q||

    è q|| = 0 (or p|| = 0)

- Gives perp transfer from ***p*** ***k***

$E(k_{||}, k_{\perp})$



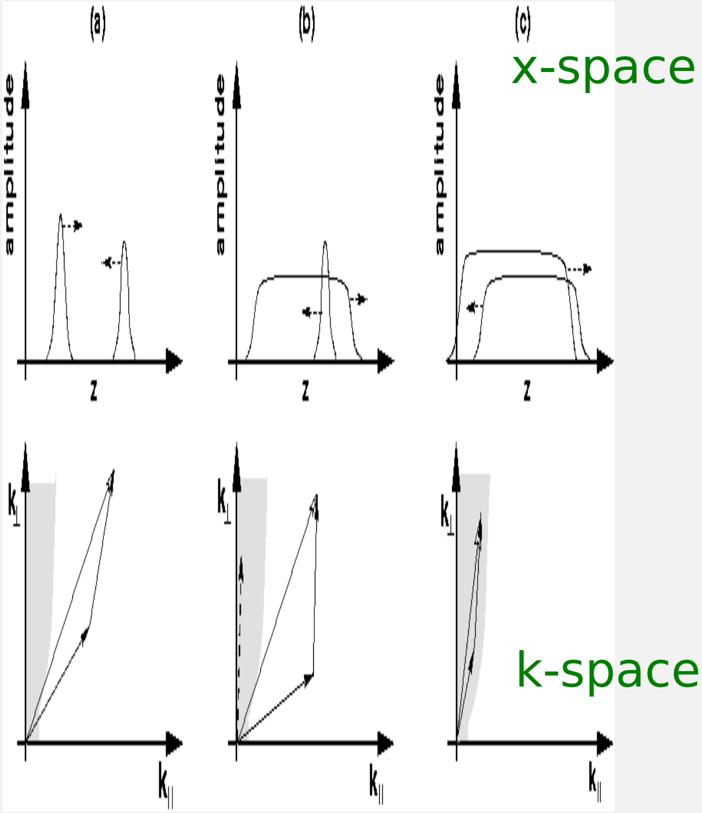
- x-space interpretation:

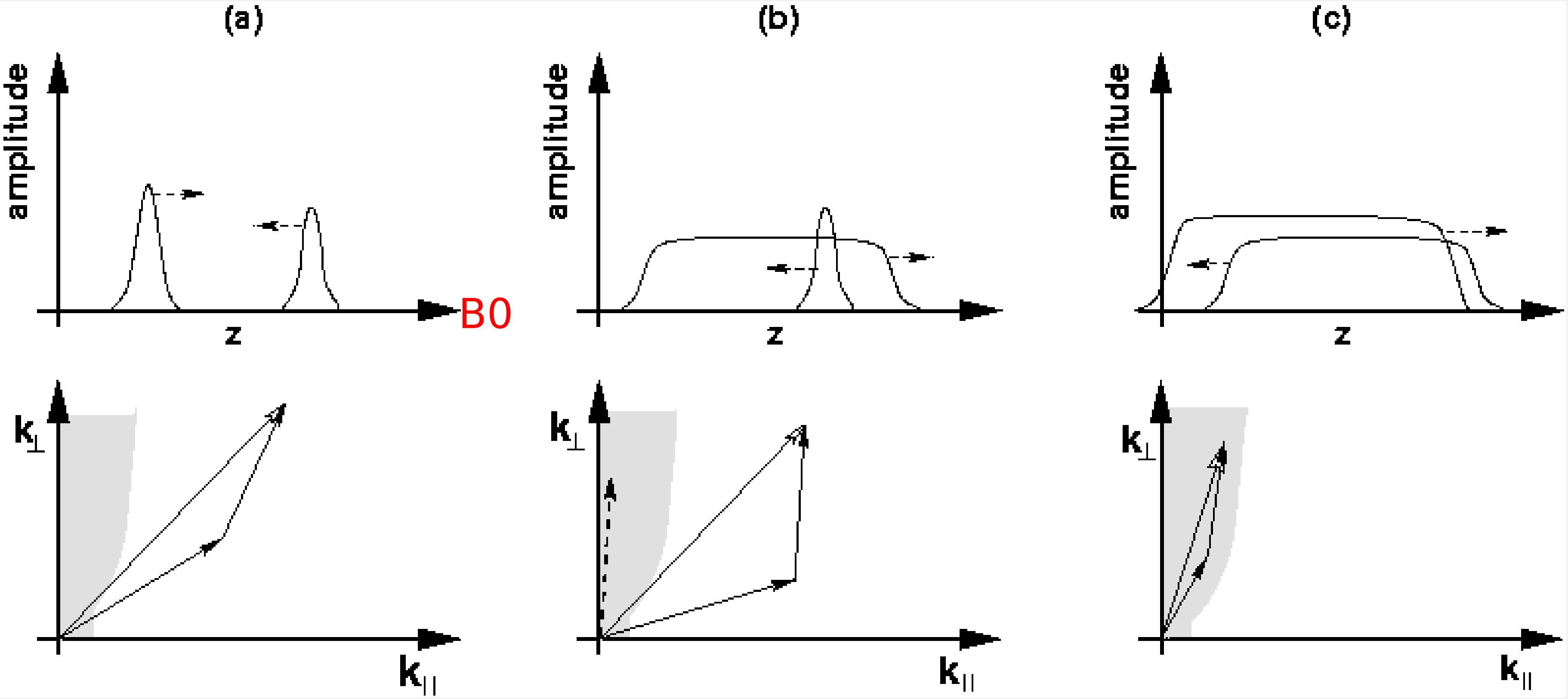
  wide/narrow wave packets

- But, more complicated than just weak turb

- Also 2 other classes of interactions

- Related to





non-resonant
wave-wave: ~Kraichnan

resonant: ~ShebalinEA83
perp transfer

‘trivially’ resonant
~hydro-like
~ unaware of B0

(Not just weak turb)

CONCEPTS:

Reduced MHD
Strauss78, Montgomery82

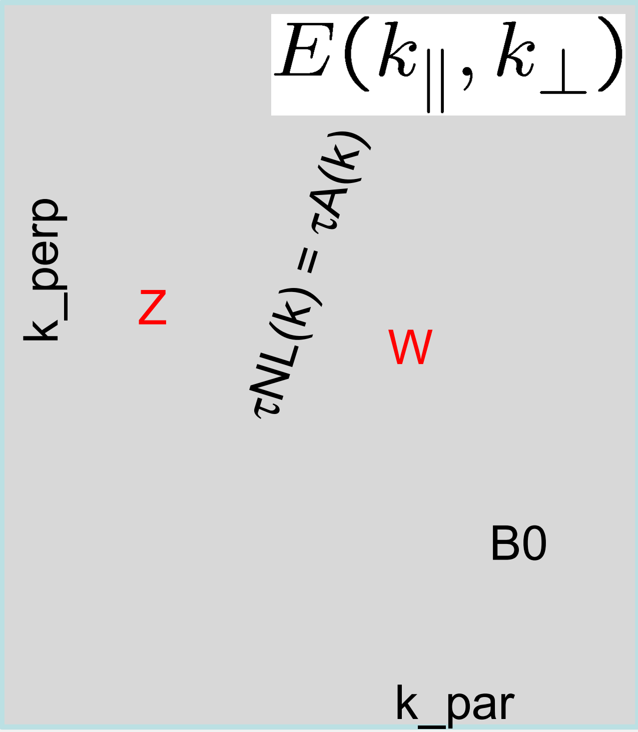
Critical Balance
Higdon86
GoldreichSridhar95

- 2 coupled components:
  - wave-like weak turb. flucts: *W*
  - quasi-2D (low-freq) turbulence: *Z*

Distinguished by which timescale

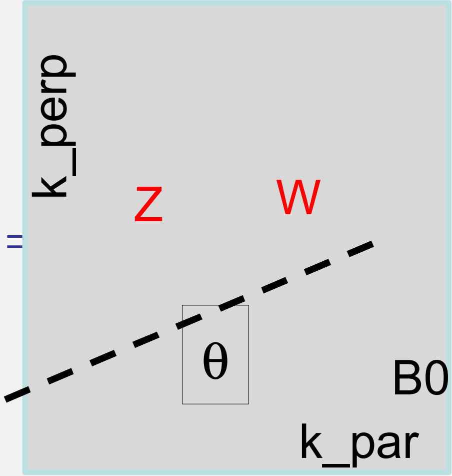
is shorter

at each ***k***: τNL(k) < τA(k)



Observ. support for ~crit balance ?

- Really want full ***k*** spectrum E(kx, ky, kz)
- 1 s/craft only gives Ered(kradial) = P(f)



- But
- by collecting data at different V- <B> angles
- can construct P(f, θ)

Vsw

θ <B>

- Ulysses ***B*** data
- wavelet `freq' spectra
- P(f, θBV)
- slope varies with θ:
-     -5/3 in ~ perp
-     -2   in || dirn

Vsw

θ <B>

Horbury et al, PRL, 2008

Studies at different times, distances:

- Horbury et al 2008
- Smith SW10 proceed.
- Podesta 2009…
- Wicks et al 2010,11

2D contrib

Slab contrib

All suggest spectra are
~critical balance style:
q-2D + wave-like
(Z) (W)

Horbury et al, PRL, 2008

NB:

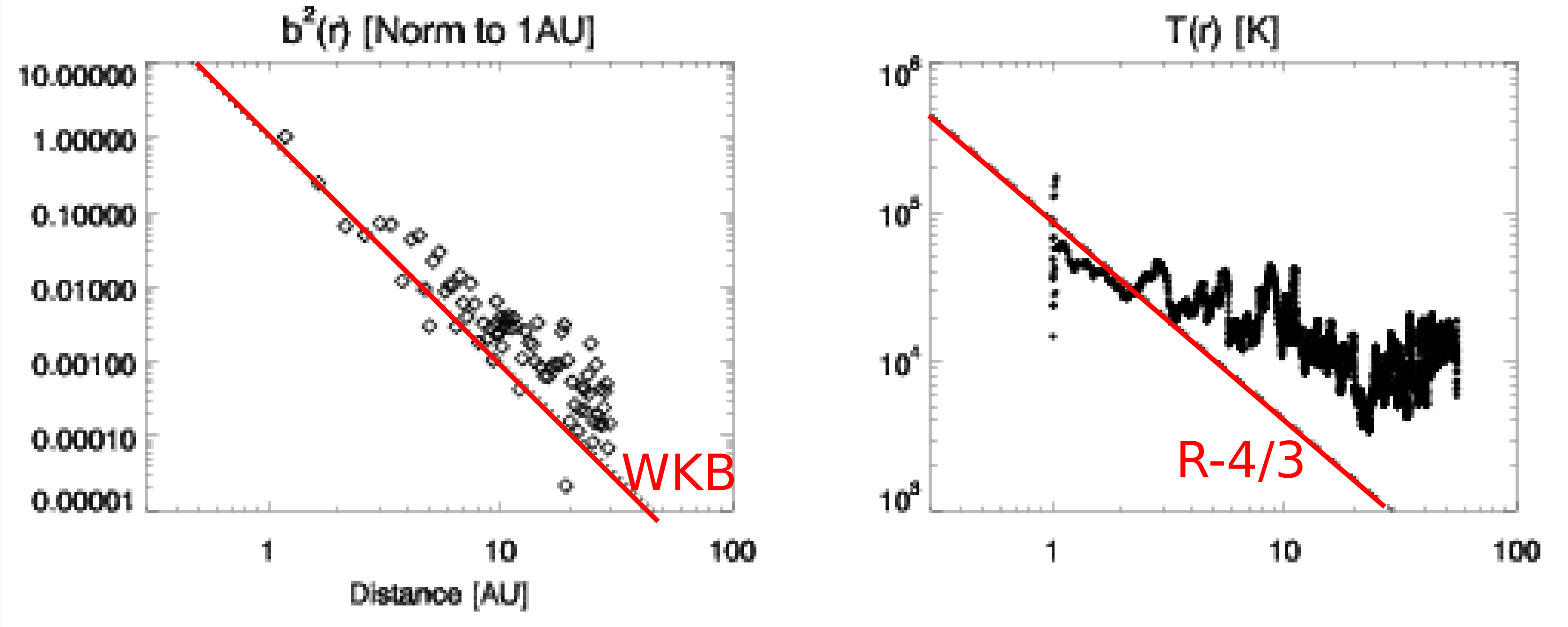
2. Studies for ***v*** not done (yet)
3. For Hc case, see,
4. eg, BeresnyakLazarian08

[redacted]

1. Observations

- Spacecraft data: fluctuations v, b, ρ

  Voyager, ACE, Ulysses, ...

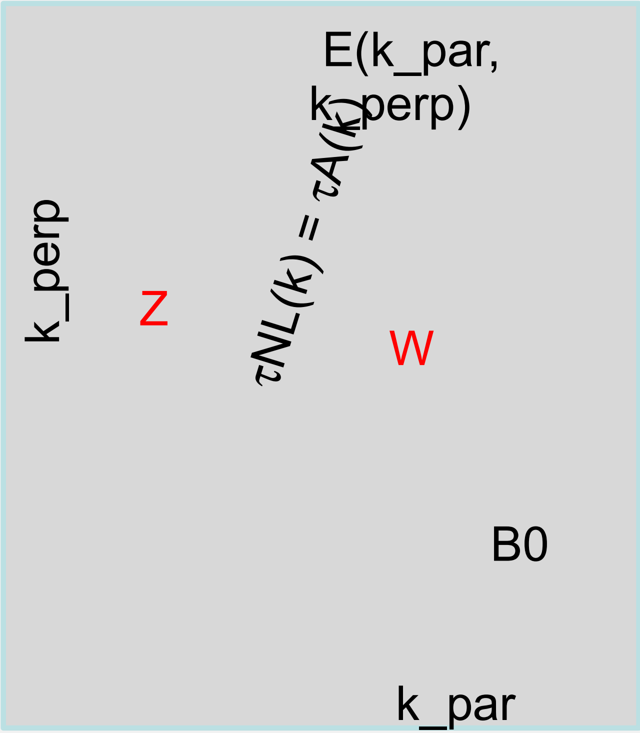


Voyager data

QSN: How do flucts evolve with distance ?

2. Objective

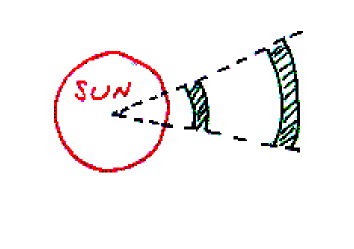
- Model radial evoln of SW flucts
- Treat flucts as 2 coupled components:
  - wave-like (high-freq) flucts: *W*
  - quasi-2D (low-freq) turbulence: *Z*



incompressible

3. Processes: What causes the evolution?

- Z,W:
  - expansion, advection
  - stream-shear [shocks, large-scale inhomog]



- W driven by pickup ions (outer heliosphere)

- Energy exchange between cpts: Z $ W

- Nonlinear cascades of W, Z proton heating

4. Why 2 components?

- Earlier transport models assumed 1 type of fluctuation

- But, physics is
  - shear drives at low-freq (non-WKB)
  - pickup ions drive high-freq flucts (Alfven waves)

  So 2 types of flucts ▪ improvement

- Equations for
  - energy, cross helicity, corrn length

  of Z and W

5. Equations: linear terms (steady)

Expansion

shear

2D energy:

$$\frac{\mathrm{d}Z^2}{\mathrm{d}r} = -\left[1 + M\sigma_D - C_{\mathrm{sh}}^Z\right]\frac{Z^2}{r}$$

wave energy:

$$\frac{\mathrm{d}W^2}{\mathrm{d}r} = -\left[1 + M\tilde{\sigma}_D - C_{\mathrm{sh}}^W\right]\frac{W^2}{r} + \frac{\dot{E}_{PI}}{U},$$

pickup ion driving

2D corrn length:

$$\frac{\mathrm{d}\ell}{\mathrm{d}r} = -\hat{C}_{\mathrm{sh}}^Z\frac{\ell}{r}$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}r} = -\hat{C}_{\mathrm{sh}}^W\frac{\lambda}{r}$$

wave corrn

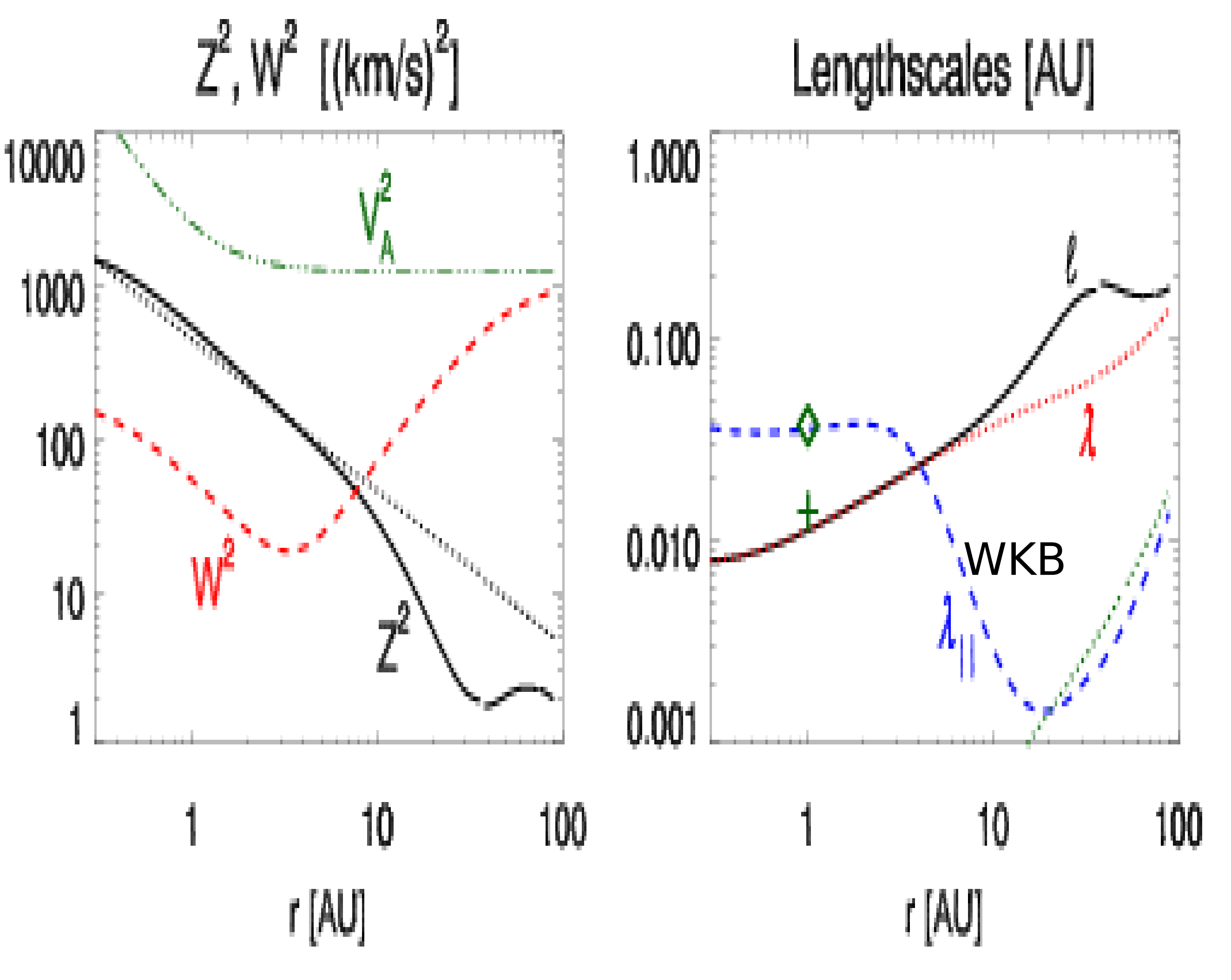
$$\frac{\mathrm{d}\lambda_\|}{\mathrm{d}r} = -\hat{C}_{\mathrm{sh}}^W\frac{\lambda_\|}{r} - \left(\lambda_\| - \lambda_{res}\right)\frac{\dot{E}_{PI}}{UW}$$

relax to λres

6. Sample Solutions

Sample solutions ...

Fixed BCs

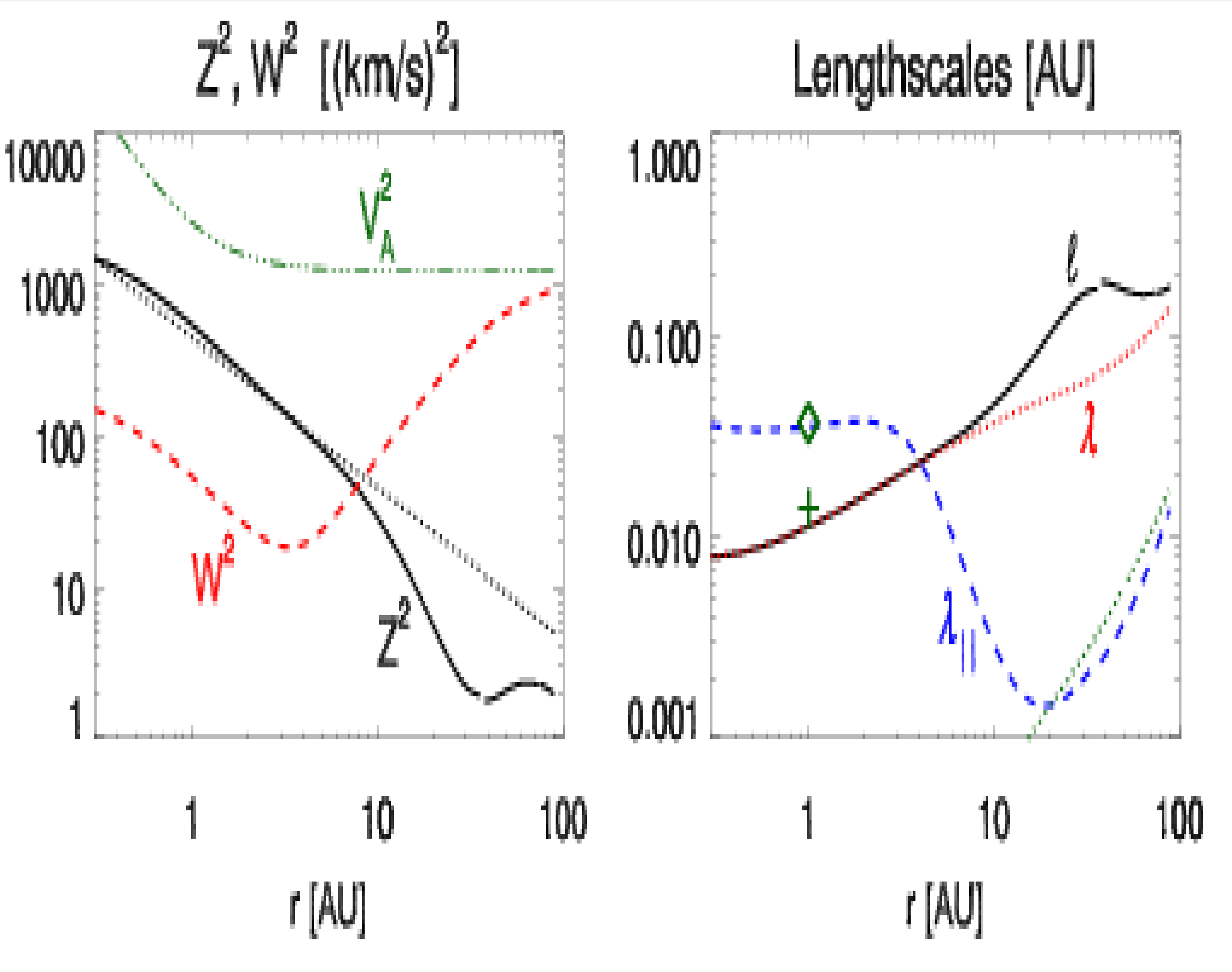


Cshear = 1

α = 2β = 0.25

σD = -1/3

Sample solutions ...



Cshear = 1

α = 2β = 0.25

σD = -1/3

+ WeygandEA09 Cluster data

Model with *Voyager* data

- ‘mapped’ solutions: different BCs for each *Voyager* interval
-
- *Voyager* data: Chuck Smith, John Richardson

distance, AU

Observ/Model agreement is encouraging

SUMMARY

SW fluctuations

can model as 2 types: wave-like + quasi-2D turb

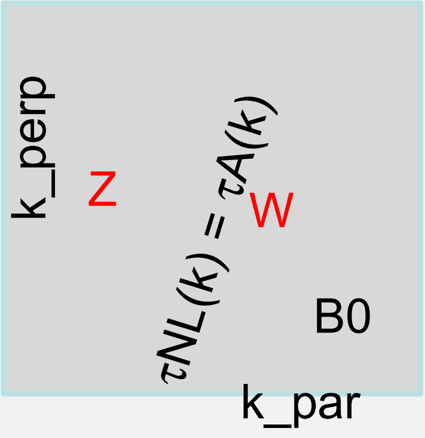
- Allows driving physics to be included more consistently.

- ~agreement with observations

- Corrn lengths:

  perp: 2-cpt model ~better fit

  than 1-cpt



Thank you

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Shortcomings

- Main weakness
  - single lengthscale for each cpt

    eg, $\lambda = \lambda_+ = \lambda_-$
  - Should really have § lengthscales for Z, W.

- Is driving of pickup ion *lengthscale* OK ?
- No compressible flucts

Nonlinear terms: Modeling

- Use 2-cpt phenom [OughtonEA06, PhysPlas] .
- Strong Va limit
- Leading-order terms are **?** cascades

  (resonant interaction with quasi-2D cpt)

- Look at zero cross helicity version first

Nonlinear terms:

? cascades

- ~von Karman-Howarth phenomenology

2D:

waves:

$$\frac{\mathrm{d}Z^2}{\mathrm{d}t} = -\frac{Z^3}{\ell} - \frac{WZ^2}{\ell}\frac{2}{1+Z/W} + X$$

$$\frac{\mathrm{d}W^2}{\mathrm{d}t} = -\frac{ZW^2}{\lambda}\frac{2}{1+\lambda/\ell} - \frac{2W^4}{\lambda^2}\frac{\lambda_{\|}}{V_a} - X$$

- Cascades proton heating
- Also eqns for lengths:

~Kraichnan

$$\ell, \lambda, \lambda_{\|}$$

Oughton et al 06, Phys. Plasma

Hc $\neq$ 0

- Same structure as Hc = 0 case
- (but uglier)

- Roughly,
- non-linear terms weakened as σ_c ▪ § 1.

eg,

$$\alpha \frac{Z^3}{\ell} \rightarrow \alpha \frac{Z^3}{\ell} f(\sigma_c)$$

$$f(\sigma_c) = \frac{\sqrt{1-\sigma_c^2}}{2}\left[\sqrt{1+\sigma_c} + \sqrt{1-\sigma_c}\right]$$

, | f| < 1

Cross helicity

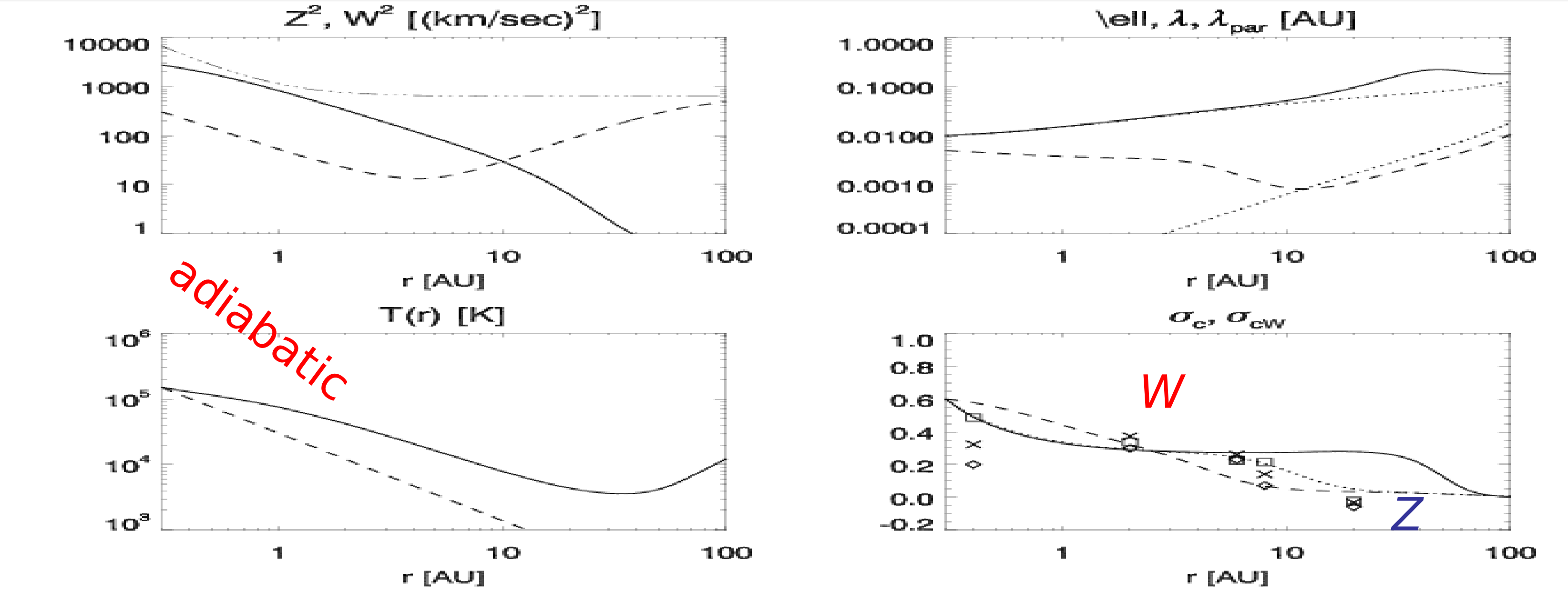
$$\frac{\mathrm{d}\sigma_c}{\mathrm{d}r} = -\left[\frac{C_{\mathrm{sh}}^{Z} - M\sigma_D}{r}\right]\sigma_c,$$

$$\frac{\mathrm{d}\tilde{\sigma}_c}{\mathrm{d}r} = -\left[\frac{C_{\mathrm{sh}}^{W} - M\tilde{\sigma}_D}{r} + \frac{\dot{E}_{PI}}{UW^2}\right]\tilde{\sigma}_c$$

proton temperature

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{4}{3}\frac{T}{r}$$

Temperature Cross



Data: Helios, Voyager

6. Steady-state equations

$$\frac{\mathrm{d}Z^2}{\mathrm{d}r} = -\left[1 + M\sigma_D - C_{\mathrm{sh}}^Z\right]\frac{Z^2}{r} - \frac{\alpha f(\sigma_c)}{U}\frac{Z^3}{\ell} + f_X\frac{X}{U}$$

$$\frac{\mathrm{d}W^2}{\mathrm{d}r} = -\left[1 + M\tilde{\sigma}_D - C_{\mathrm{sh}}^W\right]\frac{W^2}{r} - \frac{\tilde{\alpha}\tilde{f}}{U}\frac{W^2 Z}{\lambda}\frac{2}{1+\lambda/\ell} - f_X\frac{X}{U} + \frac{\dot{E}_{PI}}{U},$$

$$\frac{\mathrm{d}\ell}{\mathrm{d}r} = \left[M\sigma_D - \widehat{C}_{\mathrm{sh}}^Z\right]\frac{\ell}{r} + \beta_z\left[Z - f_X\frac{X}{U}\frac{\ell}{Z^2}\right]$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}r} = \left[M\tilde{\sigma}_D - \widehat{C}_{\mathrm{sh}}^W\right]\frac{\lambda}{r} + \beta_W\left[Z + f_X\frac{X}{U}\frac{\lambda}{W^2}\right]$$

$$\frac{\mathrm{d}\lambda_\parallel}{\mathrm{d}r} = \left[M\tilde{\sigma}_D - \widehat{C}_{\mathrm{sh}}^W\right]\frac{\lambda_\parallel}{r} - \left(\lambda_\parallel - \lambda_{res}\right)\frac{\dot{E}_{PI}}{UW^2} + \Gamma_{Nonlinear}$$

[redacted]

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_c}{\mathrm{d}r} &= \alpha_Z f' \frac{Z}{U\ell} - \left[\frac{C^Z_{\mathrm{sh}} - M\sigma_D}{r}\right]\sigma_c, \\
\frac{\mathrm{d}\tilde{\sigma}_c}{\mathrm{d}r} &= \alpha_W \tilde{f}' \frac{Z}{U\lambda}\frac{2}{1+\lambda/\ell} - \left[\frac{C^W_{\mathrm{sh}} - M\tilde{\sigma}_D}{r} + \frac{\dot{E}_{PI}}{UW^2}\right]\tilde{\sigma}_c
\end{aligned}
$$

$$
\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{4}{3}\frac{T}{r} + \frac{m_p}{3Uk_B}\left[\alpha_Z f \frac{Z^3}{\ell} + \alpha_W \tilde{f}\frac{ZW^2}{\lambda}\frac{2}{1+\lambda/\ell}\right]
$$

proton
temperature

tubulent heating

[redacted]

- Model has various parameters, controlling
  - strength of stream-shear

    [forces energies & lengthscales]
  - pickup ion driving

    [reasonably constrained/understood. IsenbergEA03, 05]
  - local conserv laws for Z or W nonlinear dynamics

- Solns are typically stable to small changes in these params.
- Similarly for small changes in

ASIDE:

Turbulence vs Waves

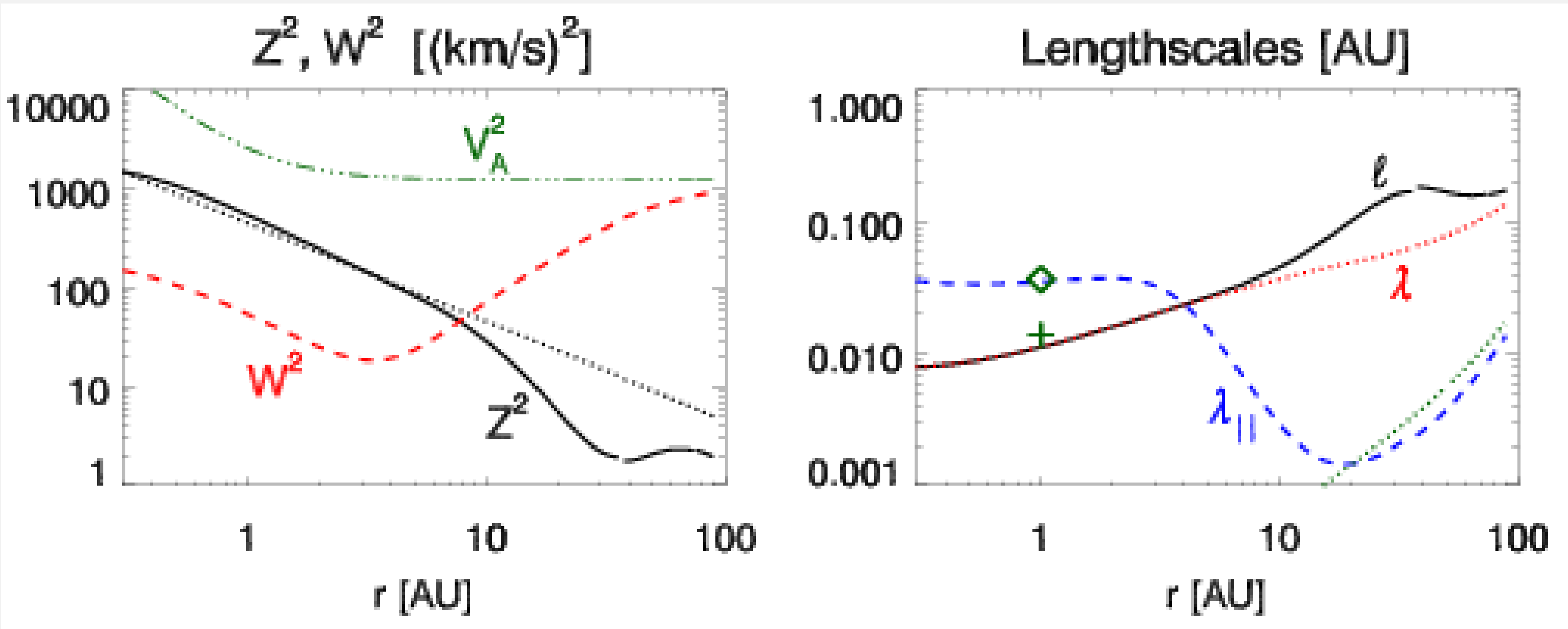
- Inherently nonlinear
  => spectral transfer
- Advection [self-distortion]
- No dispersion relation
  - each length-scale coupled to
    many time-scales (and v.v.)

- No spectral transfer (linear case)
- Propagation
- Dispersion relation ω(k)
  - each length-scale couples to
    a specific time-scale

ω(k)

k

Sample solutions ...



Csh= 1, α = 2β = 0.25,

σD = -1/3

+ WeygandEA09 Cluster data

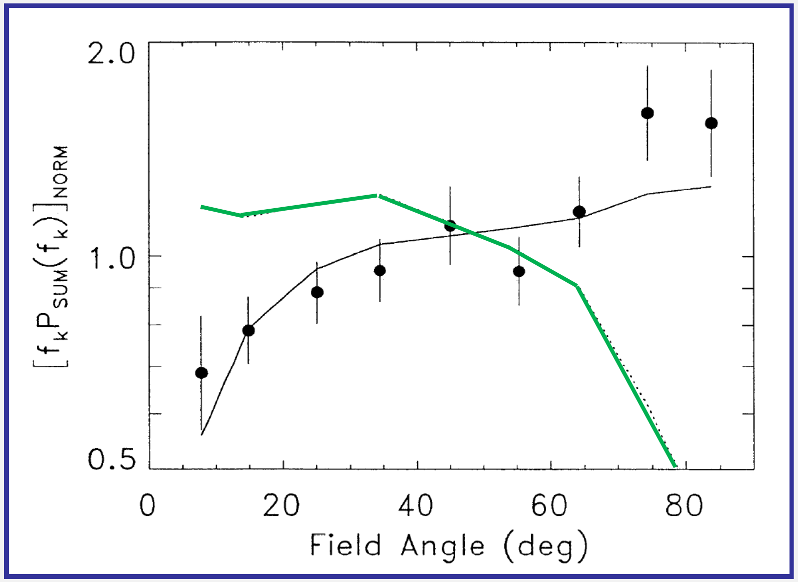
Why 2-component models ?

- Theory: RMHD, critical balance
- Simulations: GhoshEA98…
- Observational support: BieberEA94,96, …

Slab-only model

Fit: 5% slab 95% 2D

Magnetic power spectra



Bieber et al., J. Geophys. Res., 1996