Hydrodynamics

K. Scherer[1] and S.E.S. Ferreira[2]

[1]Institut für Theoretische Physik IV: Weltraum- und Astrophysik, Ruhr-Universität Bochum, Germany,

[2]North-West University, 2520 Potchefstroom, South Africa

Hydrodynamics

- General equations
- Euler equations
- Examples
- The heliosphere
  - Interaction terms
  - Pressure balance
  - Mach numbers
  - Cross sections

General equations

Continuity-, momentum-, and energy equations

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho\vec{v} \\ \rho(e+\frac{1}{2}\vec{v}^2)\end{bmatrix} + \nabla\cdot\begin{bmatrix}\rho\vec{v} \\ \rho\vec{v}\vec{v}+P\hat{I} \\ \left[\rho(e+\frac{1}{2}\vec{v}^2)+P\right]\vec{v}\end{bmatrix} = \begin{bmatrix}S \\ \rho\vec{F}+\nabla\cdot\hat{\sigma} \\ \rho\vec{v}\cdot\vec{F}+\nabla\cdot(\vec{v}\cdot\hat{\sigma})-\nabla\cdot\vec{Q}\end{bmatrix}$$

$\vec{v}$= fluid velocity

ρ = fluid density

e = internal energy of fluid

P = pressure of fluid

$\hat{I}$ = unit tensor

$\hat{\sigma}$ = viscosity/stress tensor

$\vec{F}$ = external force

$\vec{Q}$ = heat flow

S sources and sinks

Euler equations

Continuity-, momentum-, and energy equations

$\hat{\sigma}$ = $\vec{Q}$= 0 ==> Euler equations

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho\vec{v} \\ \rho(e+\frac{1}{2}v^2)\end{bmatrix} + \nabla \cdot \begin{bmatrix}\rho\vec{v} \\ \rho\vec{v}\vec{v}+P\hat{I} \\ \left[\rho(e+\frac{1}{2}v^2)+P\right]\vec{u}\end{bmatrix} = \begin{bmatrix}0 \\ \rho\vec{F} \\ \rho\vec{v}\cdot\vec{F}\end{bmatrix}$$

or in conservative form:

$$\vec{W} + \nabla \cdot \vec{f}(\vec{W}) = \vec{q}$$

Examples

1) $\nabla \cdot \vec{v} = 0$ incompressible flow (ρ = const. for stationary flow)

2) stationary 1-D spherical flow ==> $r^2 \rho v =$ const.,
and with $v =$ const ==> $\rho \sim \frac{1}{r^2}$

3) Bernoulli equation:
$\frac{1}{2}\rho v^2 + P =$ const
stationary energy equation, no forces, constant internal energy and incompressible

Multi-fluid

Continuity-, momentum-, and energy equations

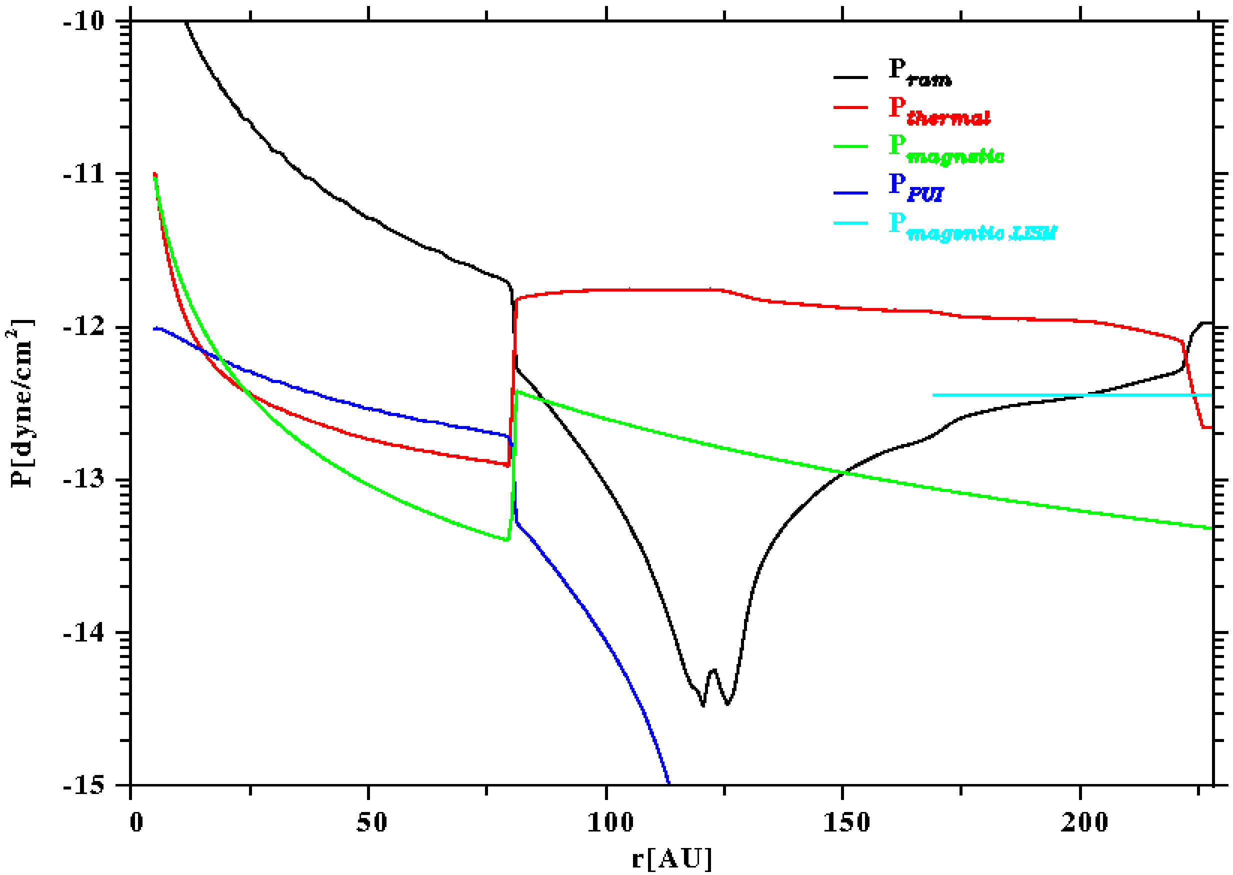
$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ \rho\vec{v}\\ \rho(e+\frac{1}{2}v^2)\end{bmatrix}+\nabla\cdot\begin{bmatrix}\rho\vec{v}\\ \rho\vec{v}\vec{v}+P\hat{I}\\ \left[\rho(e+\frac{1}{2}v^2)+P\right]\vec{u}\end{bmatrix}=\begin{bmatrix}S_c\\ \vec{S}_m\\ S_e\end{bmatrix}$$

$$S_{c,H-p}\approx\sigma(v_{rel})v_{rel}\rho_p \quad \rho_H$$

$$S_{c,H-e}\approx\sigma(E_{rel})v_{rel}\rho_p \quad \rho_H, E_{rel>13.6eV}$$

$$S_{c,H-\nu}\approx 8\cdot 10^{-8}/r^2 \quad \rho_H$$

Pressure balance



$$P_{ram} = \rho_p v_{sw}^2$$

$$P_{thermal} = \gamma_p n_p k_B T_p$$

$$P_B(< TS) = \left(\frac{B_0 r_0{}^2}{r^2} (\vec{e_r} + \frac{r\Omega}{v_{sw}} \vec{e_\theta}) \right)^2 /(8$$

$$P_B(> TS) = \left(\frac{B_0 r_0{}^2}{r^2} \right)^2 /(8\pi)$$

$$P_{PUI} = \rho_{pui} v_{sw}^2$$

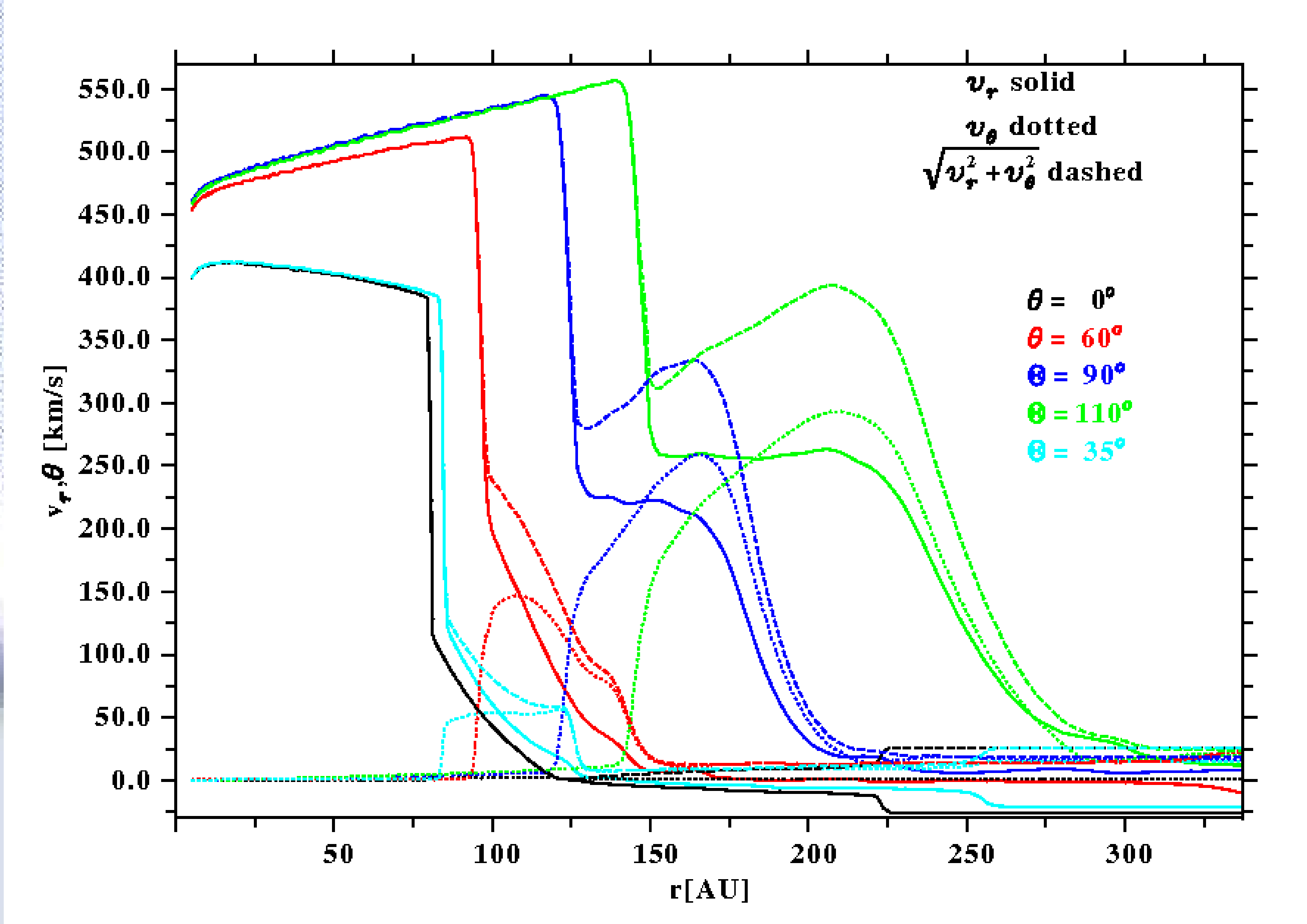
$$P_{B,LISM} = B_{LISM,0}^2 /(8\pi)$$

$$P_{CR} \approx P_{B,LISM} \approx 0.35\, eV\, cm^{-3}$$

$$B_0 = 5\mu G$$

$$B_{LISM,0} = 3\mu G \quad (1 - 10\mu G)$$

Heliosheath' proton speeds



Sound speed: Multifluid plasma

Critical point (Parker), stationary flow:

Continuity equation: $\nabla \cdot \rho_i \vec{u} = 0$

momentum equation: $\rho_i (\vec{u} \cdot \vec{\nabla}) \vec{u} = -\nabla P_i + \vec{S}_i$

For each i one gets:

$$\nabla \cdot \vec{u}(\rho_i u^2 - kT_i n_i) = -kn_i(\vec{u} \cdot \nabla T_i) + (\vec{u} \cdot \vec{S}_i)$$

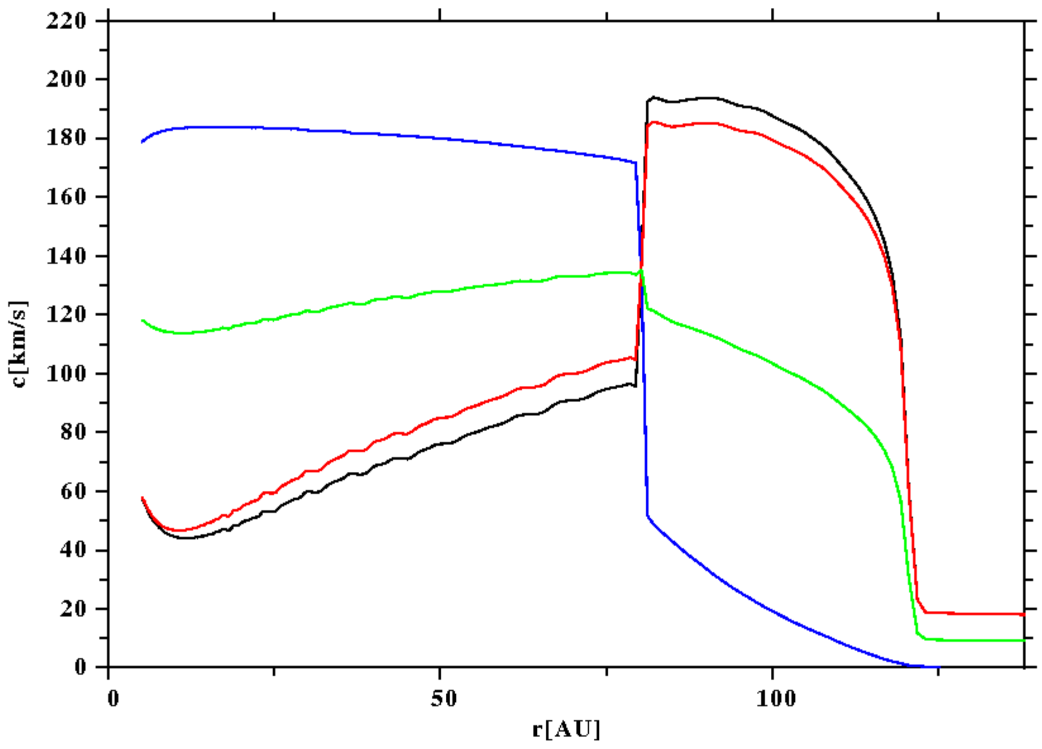
and summing up:

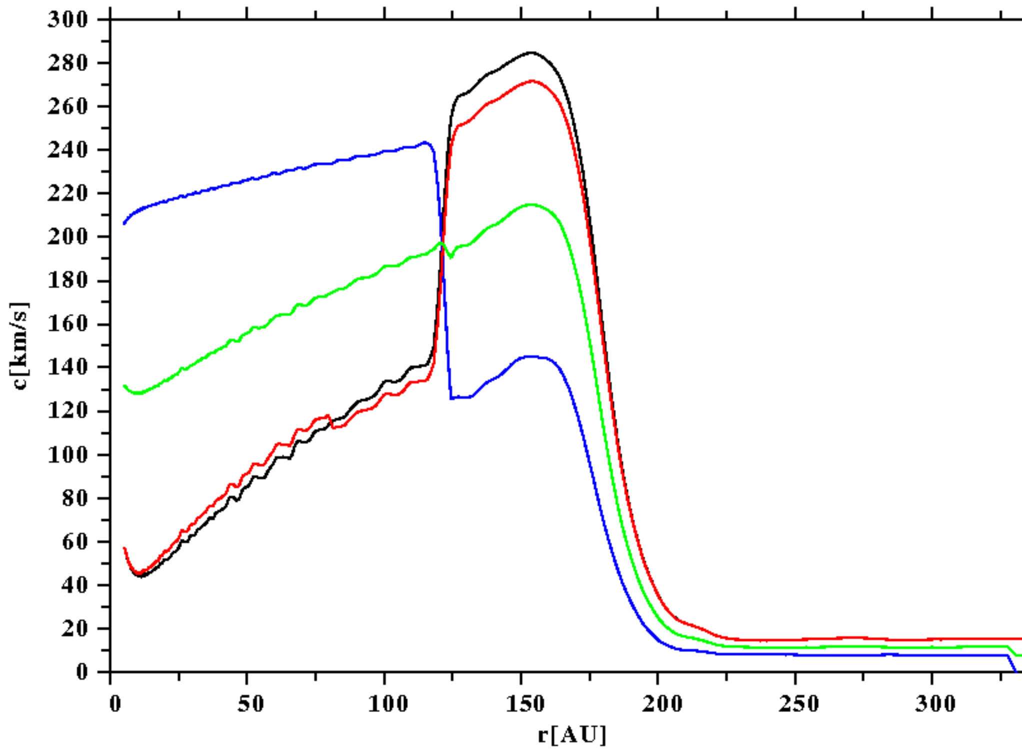
$$\nabla \cdot \vec{u} \left(\sum_i \rho_i u^2 - \sum_i kn_i T_i \right) = -\vec{u} \cdot \left(\sum_i (kn_i \nabla T_i + \vec{S}_i) \right)$$

defining the total sound (critical) speed as:

$$c_{tot} = \sqrt{\frac{\sum_i \gamma_i P_i}{\sum_i \rho_i}}$$

Sound speed





ecliptic

pole

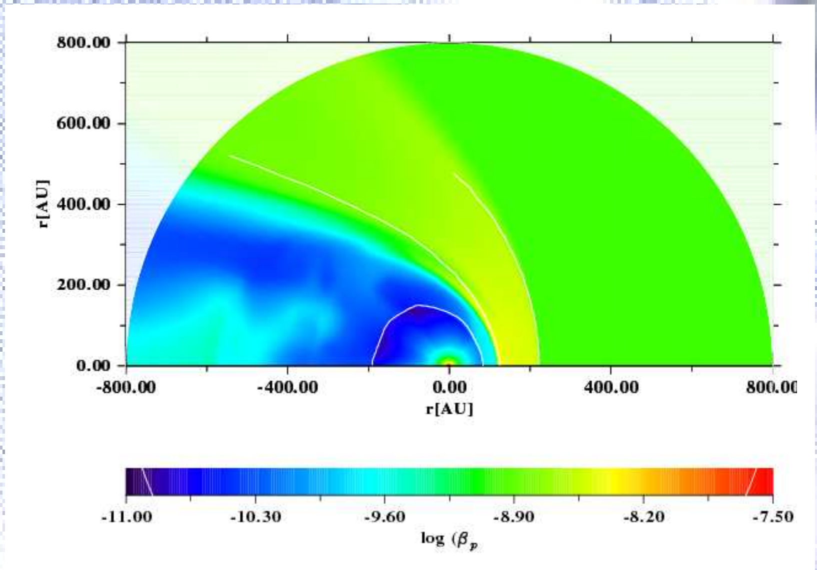
c_p, c_{pui}, c_{tot}, $(c_p + c_{pui})/2$

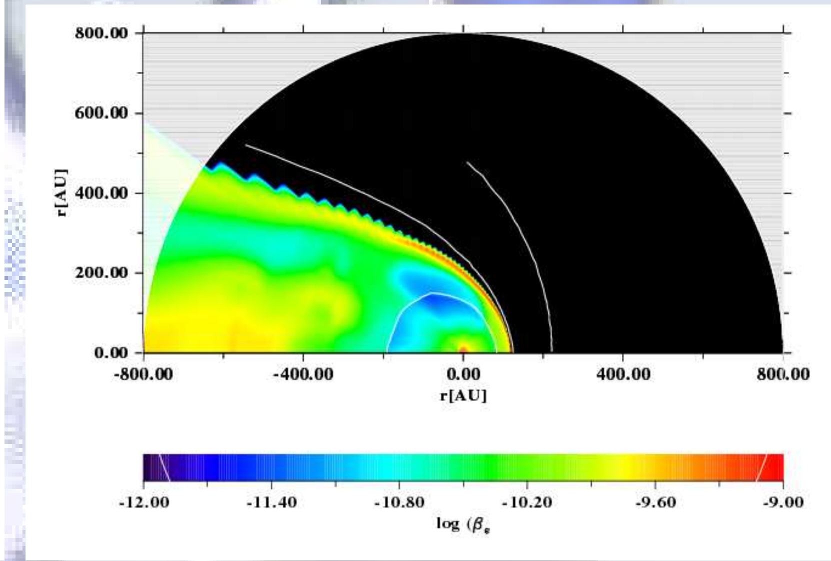
$P_{pui} = \frac{1}{5} P_{pui,ram}$

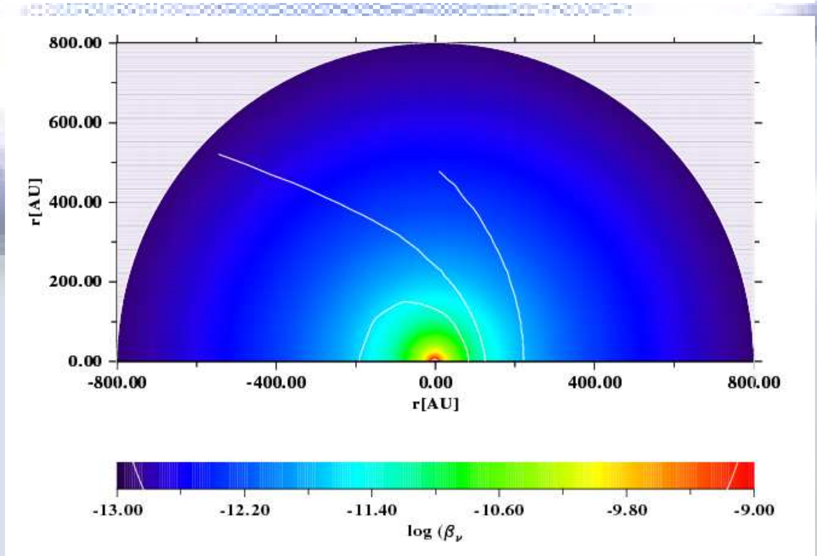
Collision rates

$$\beta_p = \sigma(v_{rel}) v_{rel} \rho_p$$

$$\beta_e = \sigma(E_e) v_{rel} \rho_{e=p}$$



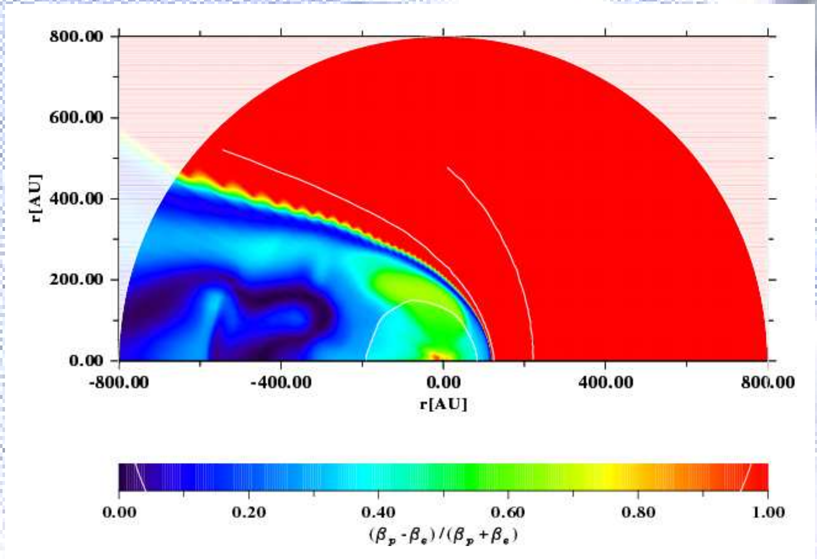




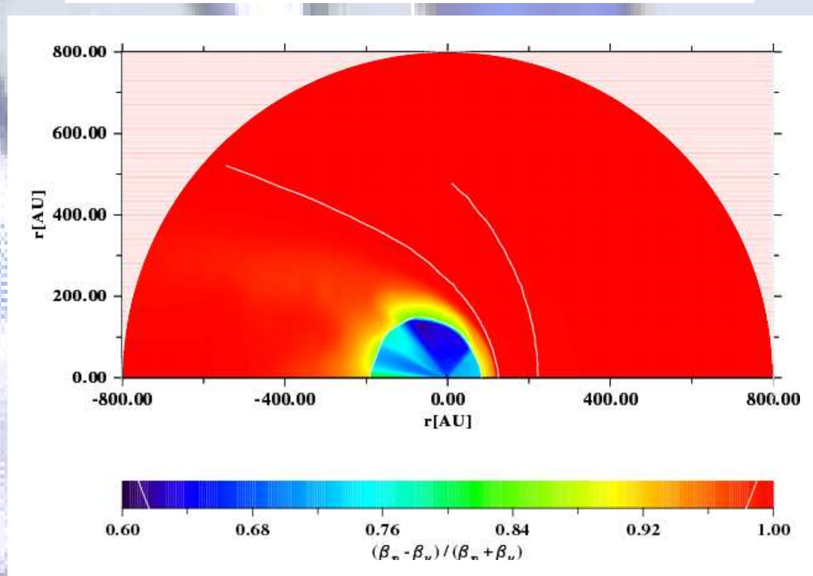
$$\beta_\nu = 8 \cdot 10^{-8} r^{-2}$$

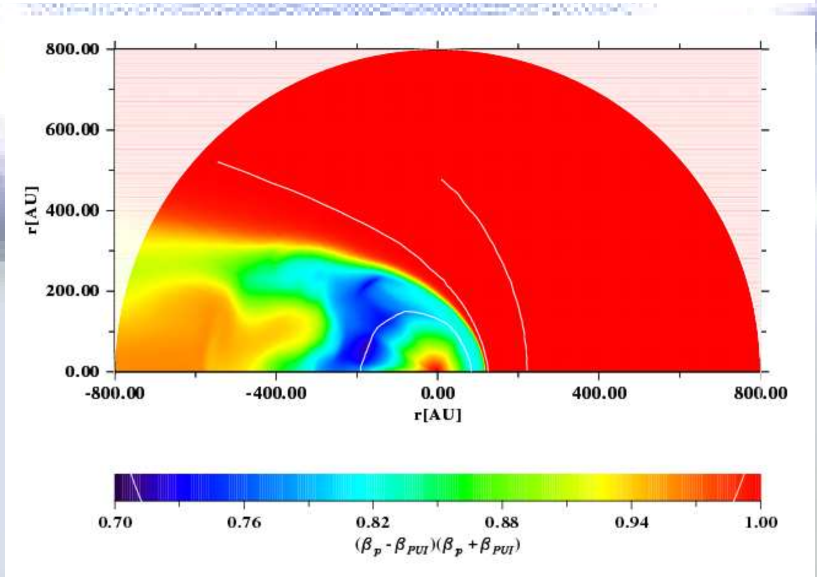
Ratios of collision rates

$$(\beta_p - \beta_e)/(\beta_p + \beta_e)$$



$$(\beta_p - \beta_\nu)/(\beta_p + \beta_\nu)$$





$$(\beta_p - \beta_{PUI})/(\beta_p + \beta_{PUI})$$

The heliospheric tail?