Modelling cosmic ray modulation by stochastic processes

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Bochum workshop

Cosmic Ray Transport

We solve *Parker (1965)* transport equation in the form

$$\frac{\partial f}{\partial t} = -\left(\vec{V}_{sw} + \vec{v}_d\right) \cdot \nabla f + \nabla \cdot (\mathbf{K} \cdot \nabla f) + \frac{1}{3}\left(\nabla \cdot \vec{V}_{sw}\right) p \frac{\partial f}{\partial p}$$

Make use of stochastic differential equations (SDEs), we solve this equation (backwards in time) in 5D.

Assuming $\rho \propto f$ we use the time backwards Kolmogorov equation

$$\frac{\partial f}{\partial t} = \sum_i \left(A_i \frac{\partial f}{\partial x_i}\right) + \frac{1}{2}\sum_{i,j}\left(B_{ij} \cdot B_{ij}^T \frac{\partial^2 f}{\partial x_i \partial x_j}\right)$$

to derive the appropriate set of SDEs, which is then solved numerically.

SDE models discussed by e.g. *Strauss et al. (2011) ApJ*, *Zhang (1999)*, *Alanko-Huotari et al. (2007)*, ... and lots more ...

Stochastic integration ...

This transforms the TPE into a set of $N-1(=4)$ 2D differential equations

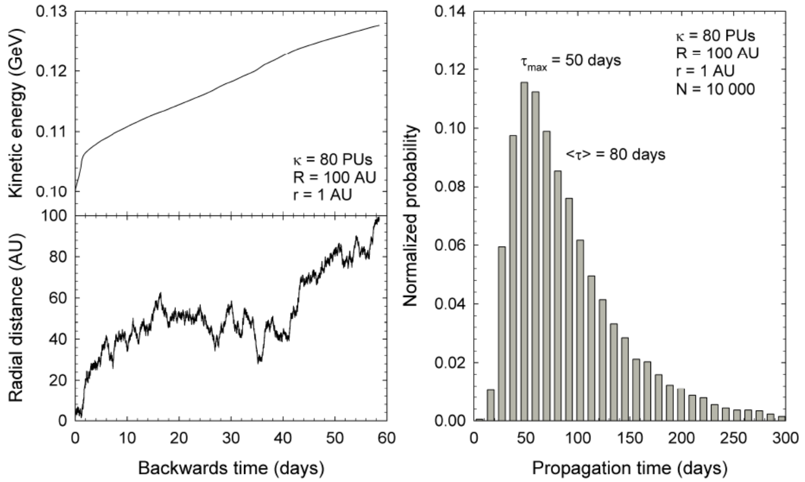
$$d\vec{x} = \vec{A}dt + \mathbf{B} \cdot d\vec{W}$$

with e.g. $x_i \in \{r, \theta, \phi, E\}$ and $d\vec{W} = \vec{\eta}\sqrt{dt}$.
These are solved (simultaneously) numerically be an Euler scheme $(\Delta t \sim dt)$

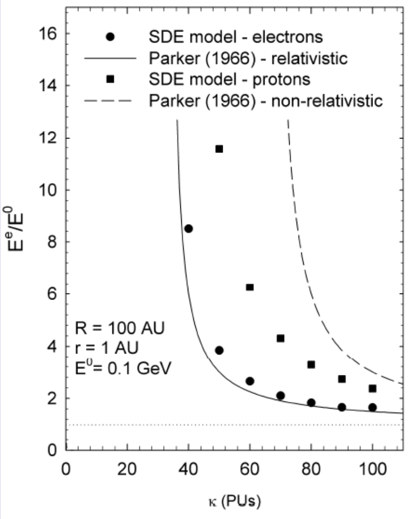
$$\vec{x}^t = \vec{x}^{t-1} + \Delta\vec{x}^{t-1}$$

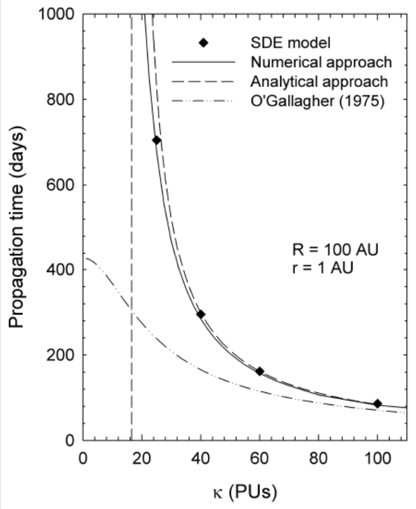
with some starting point $\vec{x}^0$.
This pseudo-particle is traced up to a modulation boundary (when using backwards in time; stationary solution). Averaging a lot of them, we can calculate e.g. j at $\vec{x}^0$.

Calculation of propagation times and energy losses



Propagation times and energy losses – 1D solutions





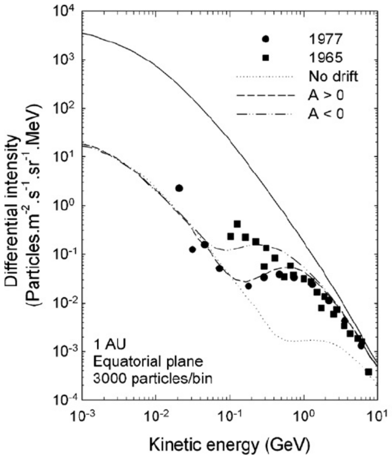
Electron modulation model setup ...

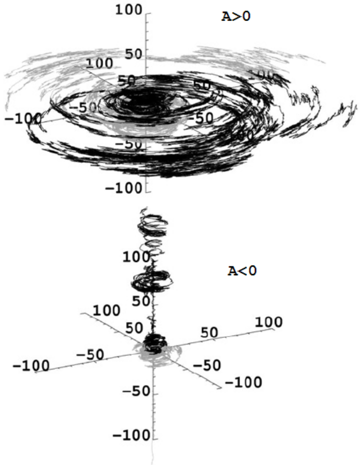
- Solve for galactic and Jovian electrons simultaneously.
- We add the Jovian magnetosphere as a solid angle is space.
- Jupiter also moves time dependently along its trajectory.
- Numerically, Jupiter is added as a second modulation boundary.
- We can then calculate the total electron flux as

$$f(\vec{r}^0, p^0, t^0) = \int_{\vec{r}\in\Omega_b} f_b(\vec{r}_b)\rho(\vec{r}_b, p|\vec{r}^0, p^0)dp + \int_{\vec{r}\in\Omega_b'} f_b'(\vec{r}_b')\rho(\vec{r}_b', p|\vec{r}^0, p^0)dp.$$

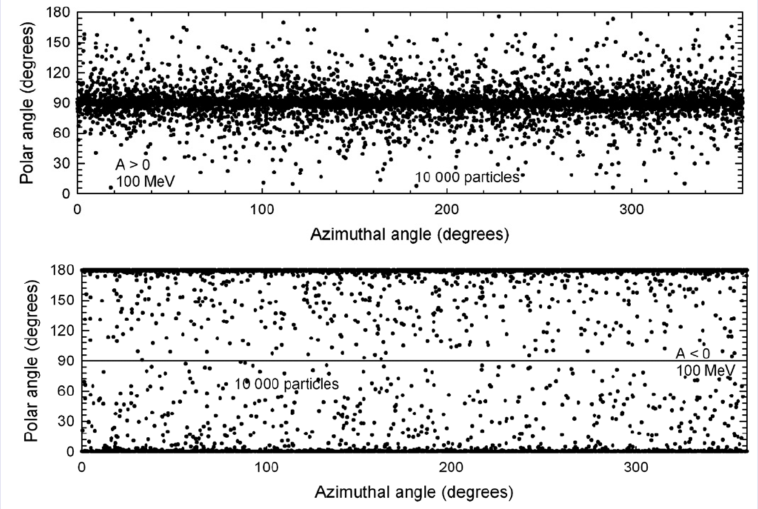
with $\Omega_b \bigcap \Omega_b' = \mathbf{0}$, leading to $j_t = j_g + j_j$.

Galactic electrons

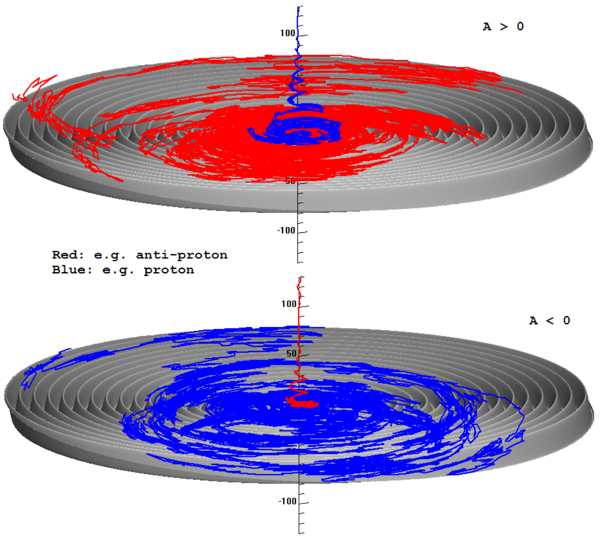




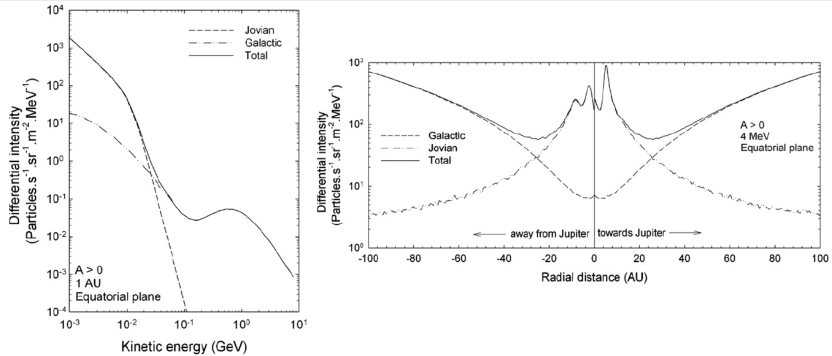
Drift visualization



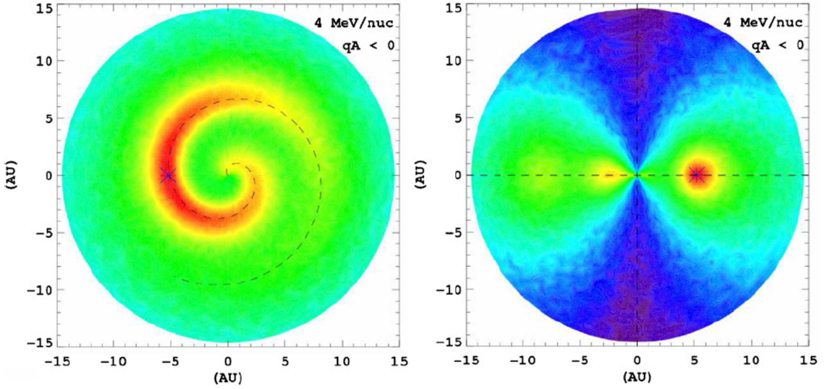
Drift visualization



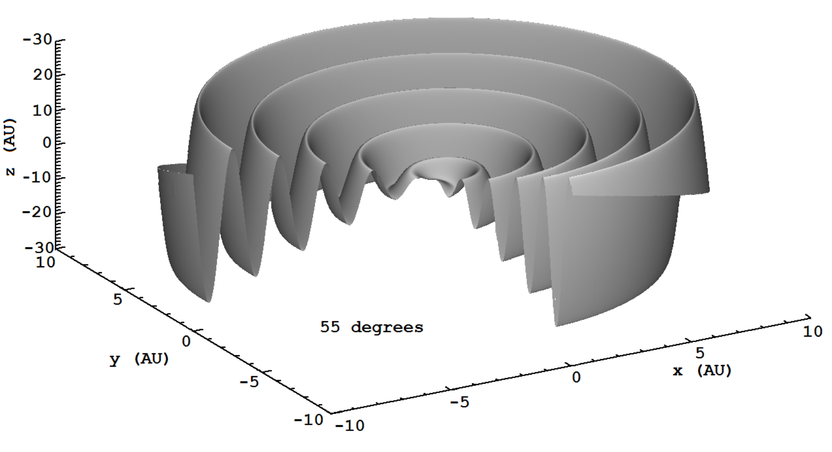
Jovian electrons



Jovian electrons

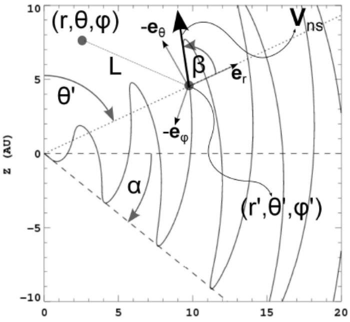


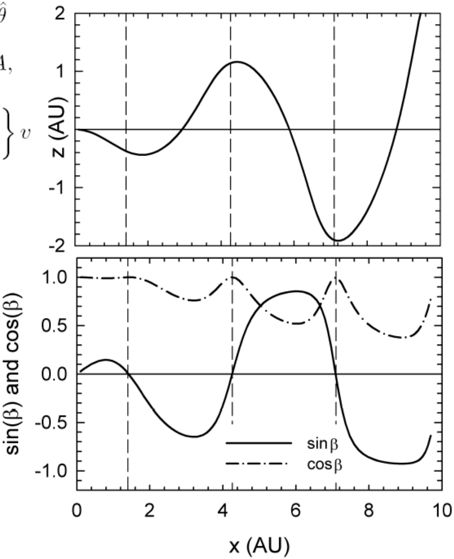
Wavy current sheet



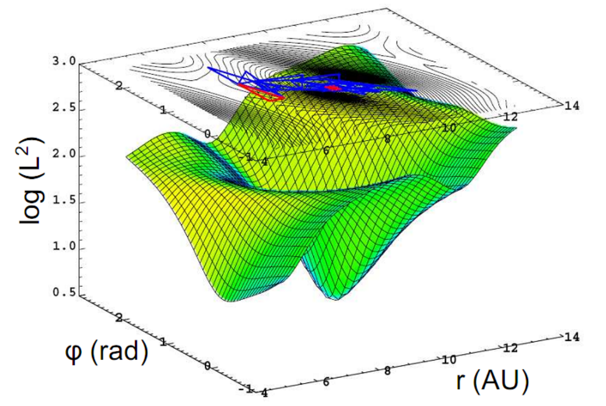
$$\vec{v}_{ns} = v_{ns} \left\{ \cos(\pm\beta) \sin\Psi \hat{r} + \sin(\pm\beta) \hat{\theta} + \cos(\pm\beta) \cos\Psi \hat{\phi} \right\} \cdot qA,$$

$$v_{ns} = \left\{ 0.457 - 0.412 \frac{|L|}{r_L} + 0.0915 \frac{|L|^2}{r_L^2} \right\} v$$





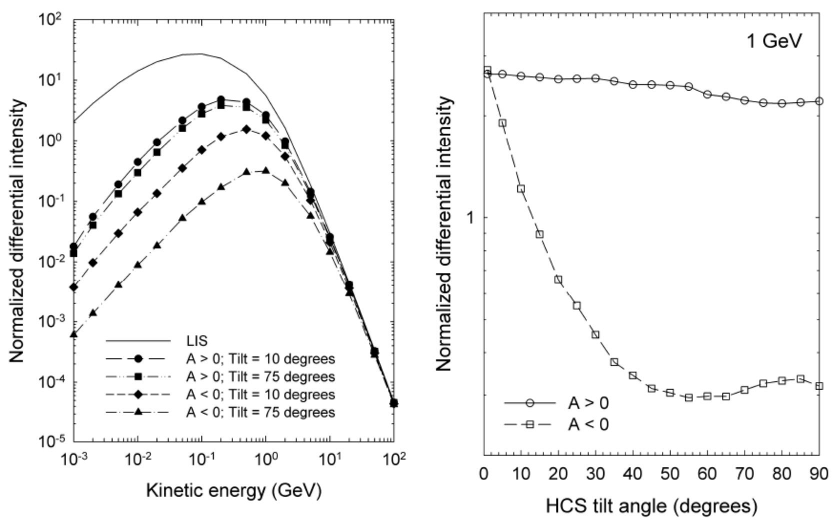
- The shortest distance to the HCS has to be obtained, i.e. minimize a line element in 2D.
- This has to be done numerically . . . Nelder-Mead method.



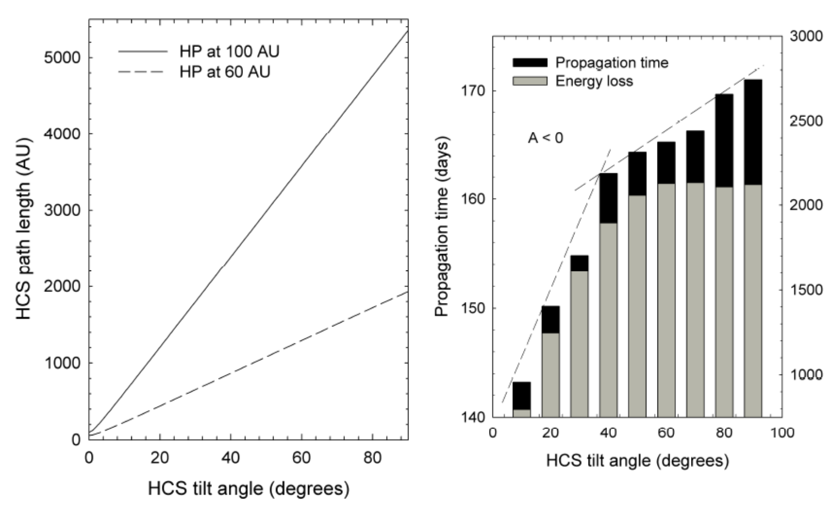
$$L^2 = (r - r')^2 + r^2(\theta - \theta')^2 + r^2 \sin^2\theta(\phi - \phi')^2$$

but with $\theta' = \theta'(r', \phi')$.

Drift results



The HCS *stickyness*



The HCS *stickyness*