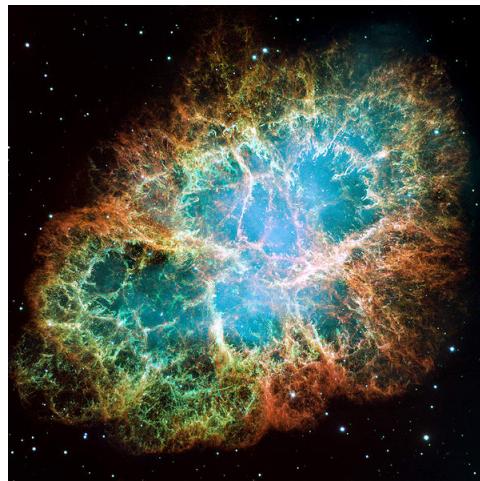
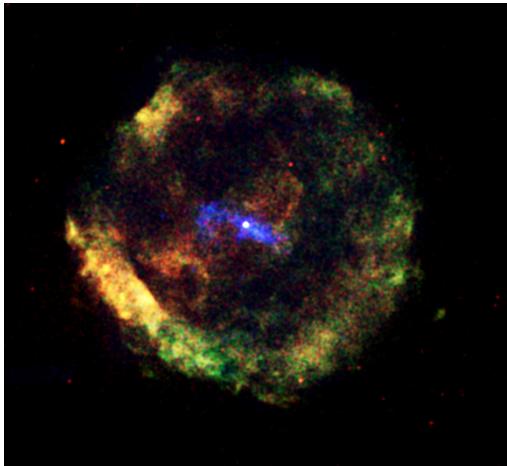


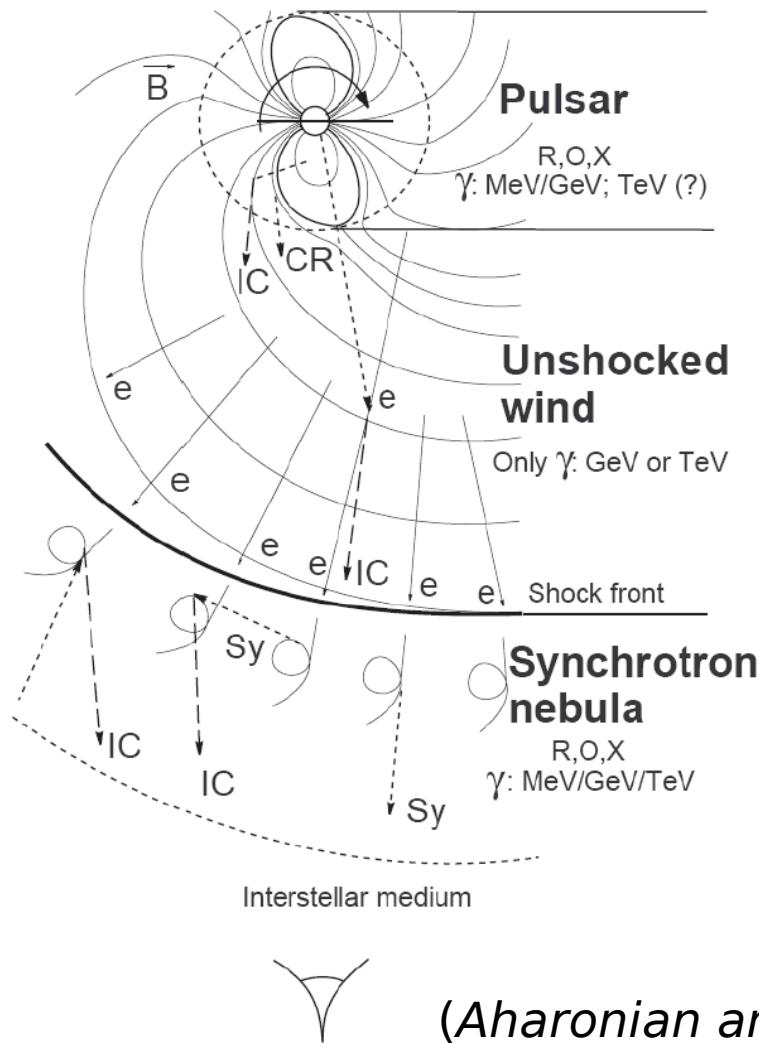
# MODELLING THE EVOLUTION OF PULSAR WIND NEBULA



M.J. Vorster

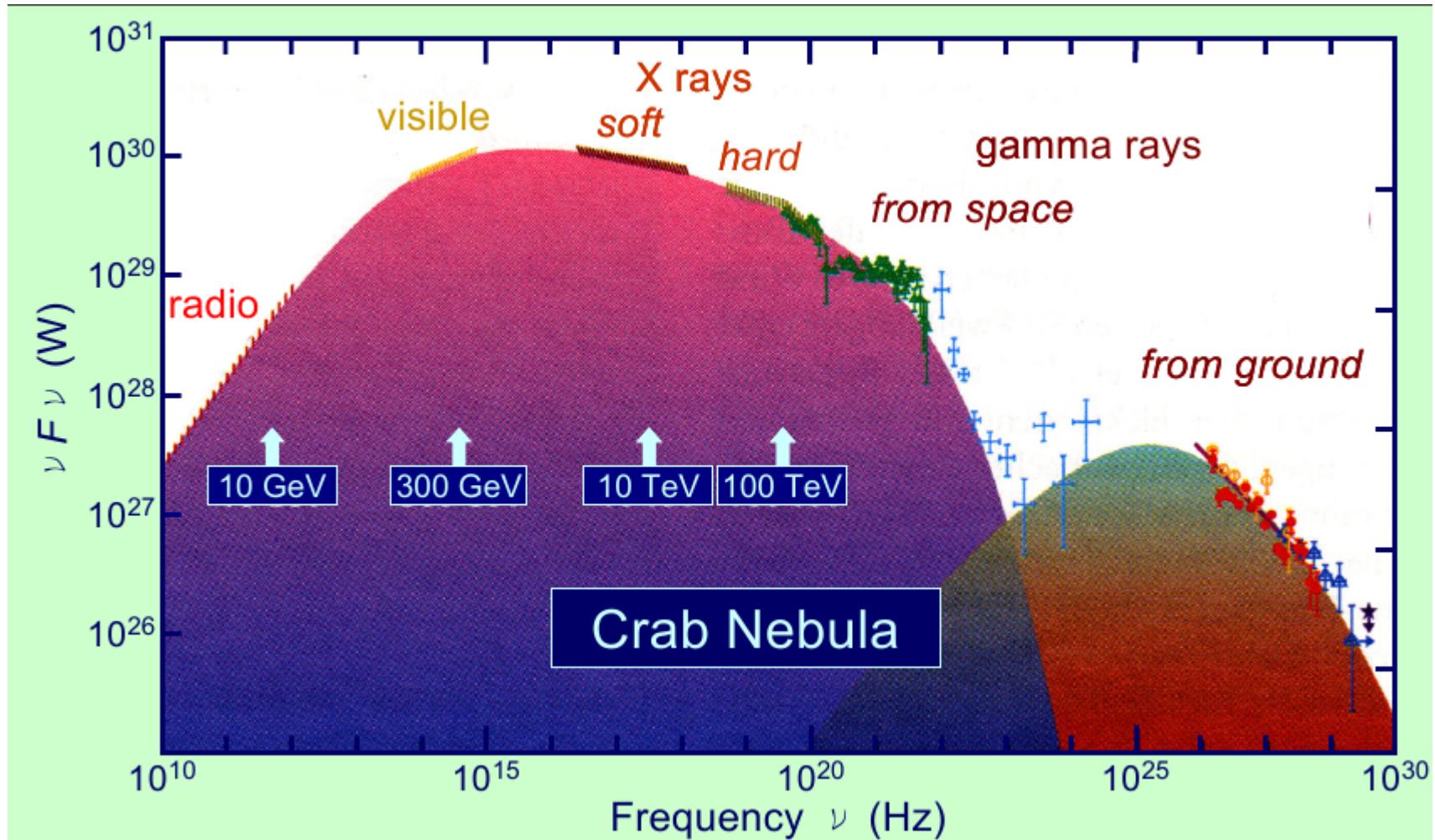
Supervisors: S.E.S. Ferreira, H. Moraal

# Pulsar Wind Nebulae

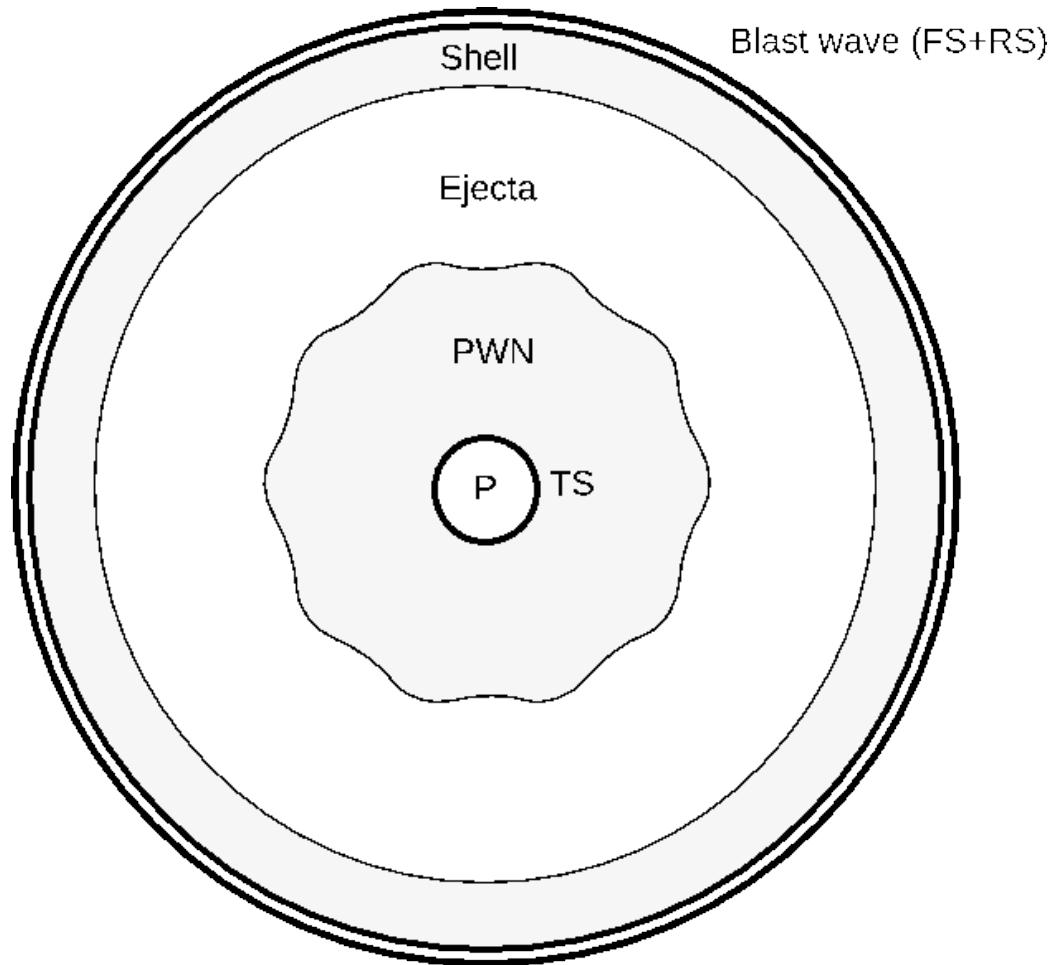


(Aharonian and Bogovalov, 2003)

# Non-thermal spectra



# Evolution of PWN'e



# (Magneto)hydrodynamic Modelling

- Euler equations

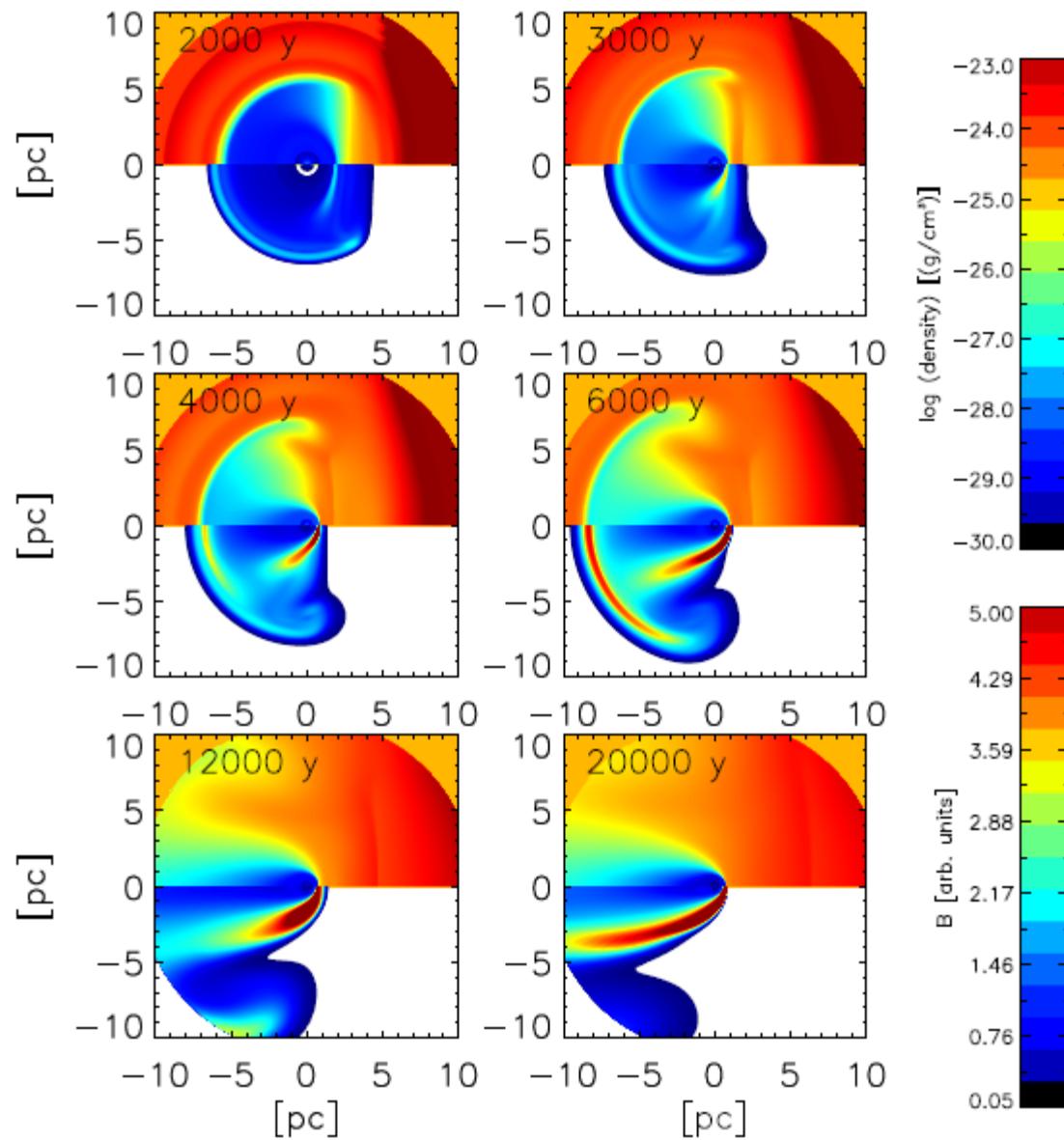
$$\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \mathbf{v}) = 0,$$

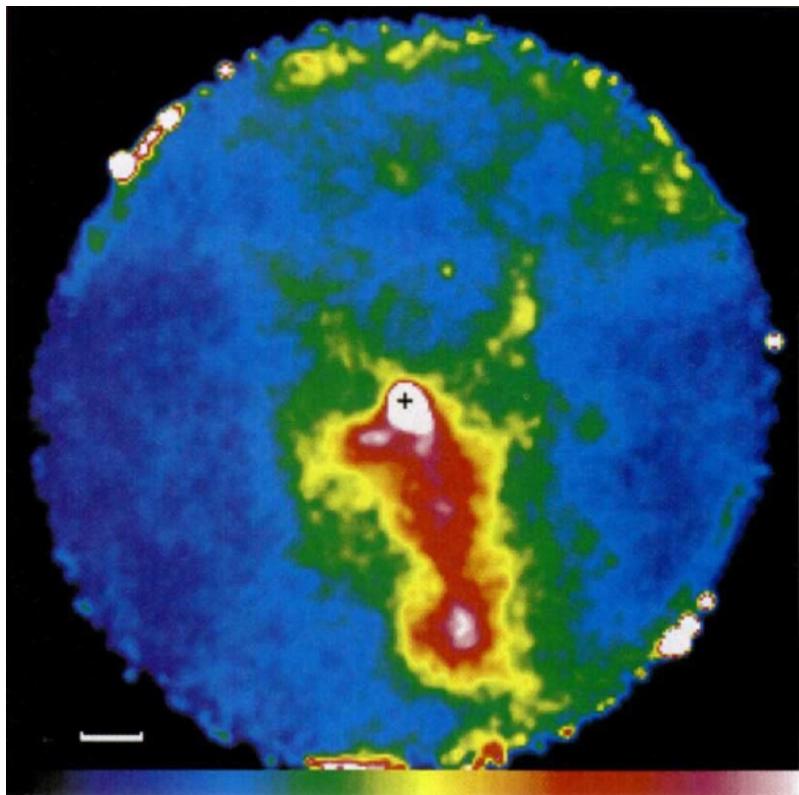
$$\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v} + P \mathbf{I}) = 0,$$

$$\frac{\partial}{\partial t} \left( \frac{\rho}{2} \mathbf{v}^2 + \frac{P}{\gamma - 1} \right) + \nabla \cdot \left( \frac{\rho}{2} \mathbf{v}^2 \mathbf{v} + \frac{\gamma \mathbf{v} P}{\gamma - 1} \right) = 0$$

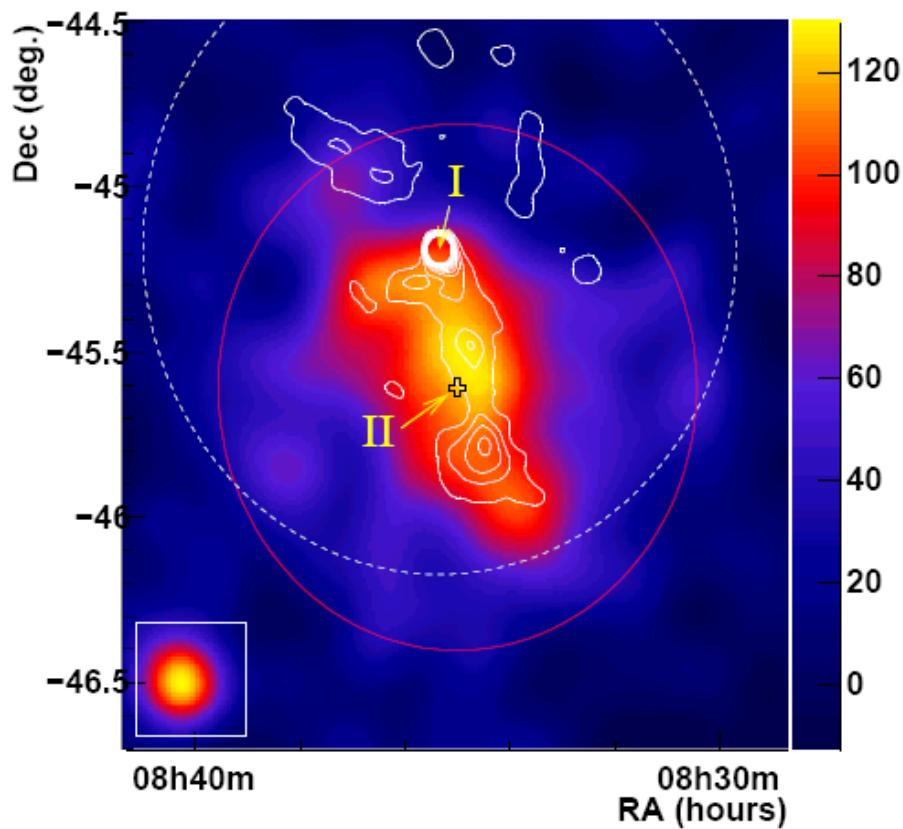
- Kinematic B-field

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B}) = 0$$



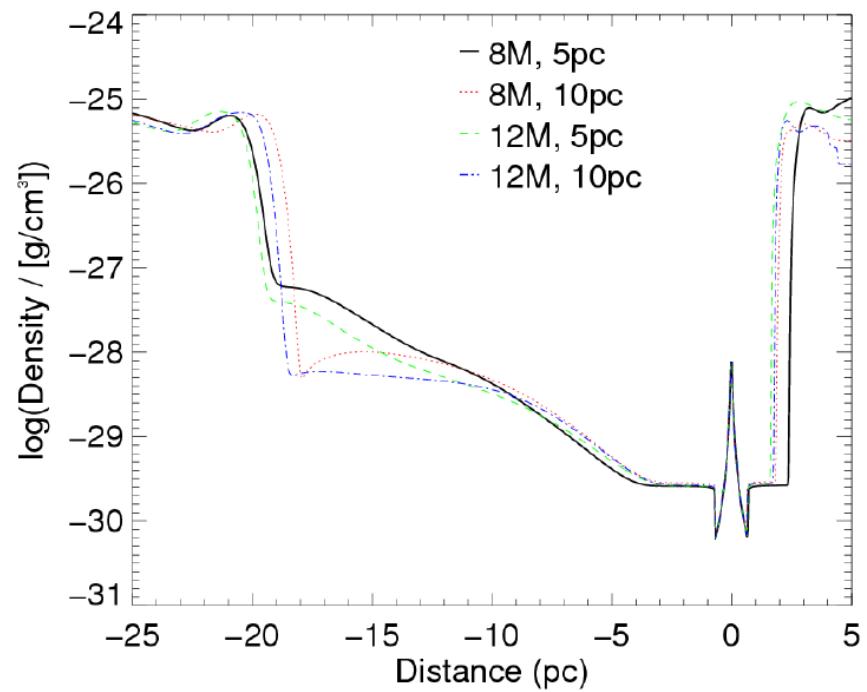
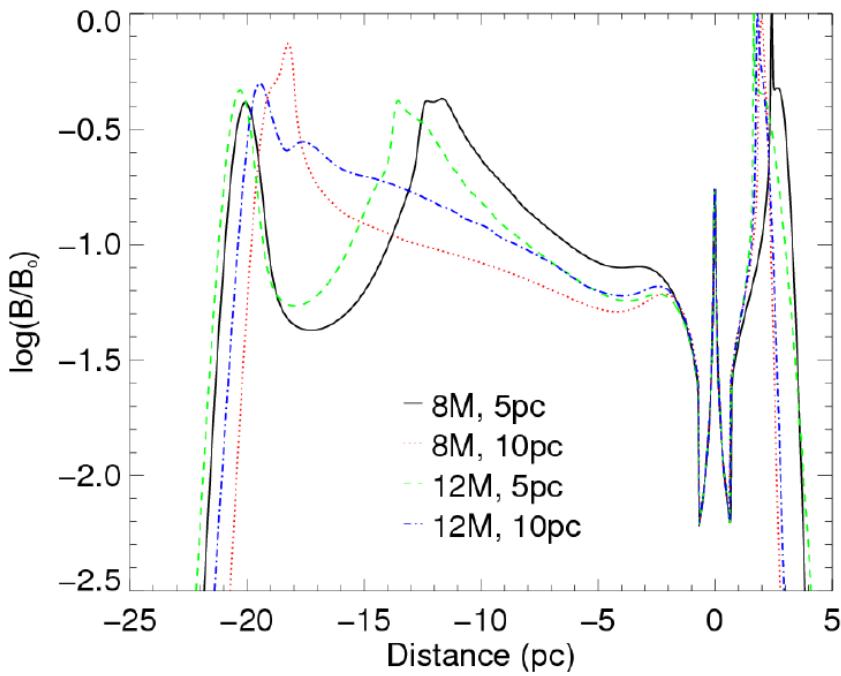


*(Markwardt and Ögelman,  
1995)*



*(Aharonian et al.,  
2006)*

## *Non-uniform evolution: radial profiles after 12 000 years*



# The Particle Transport Equation

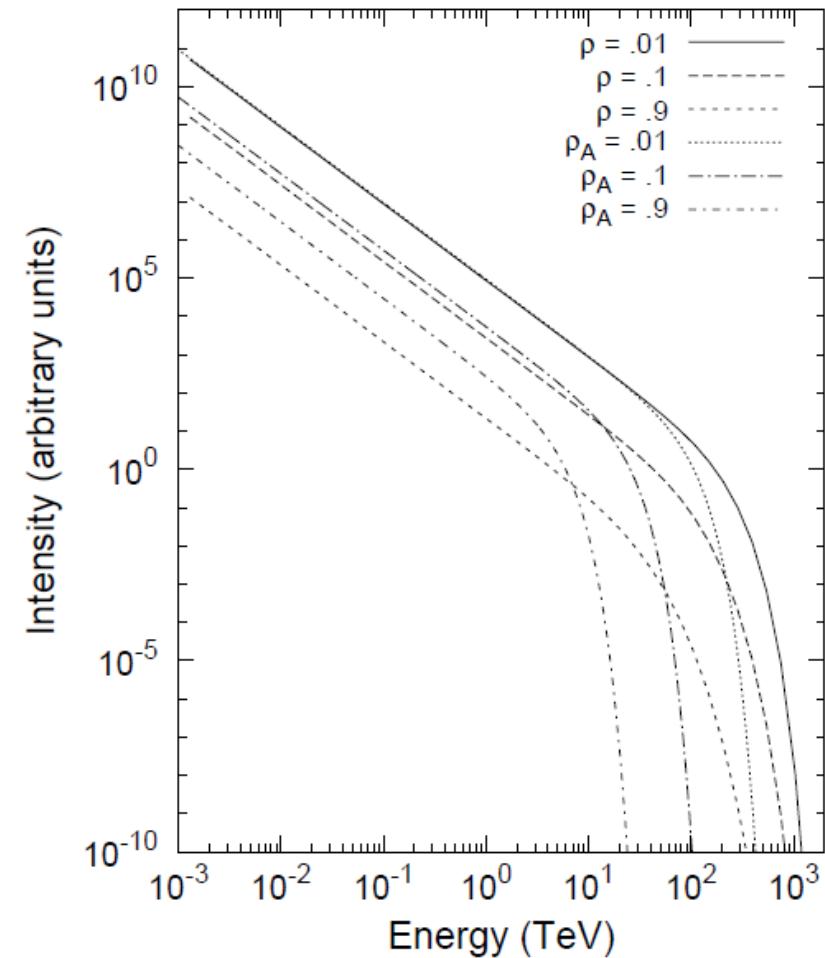
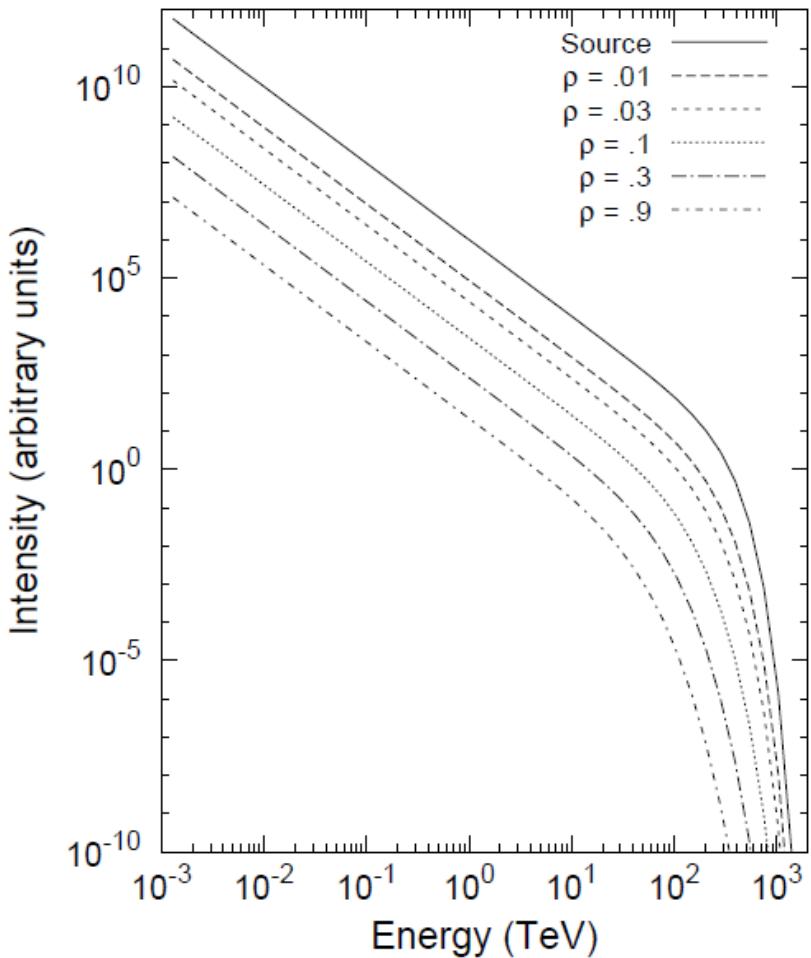
$$\frac{\partial f}{\partial t} - Q = \nabla \cdot [\underline{\mathbf{K}} \cdot \nabla f - \mathbf{v} f] + \frac{1}{p^2} \frac{\partial}{\partial p} \left[ \dot{p} p^2 f + \frac{(\nabla \cdot \mathbf{v})}{3} p^3 f \right]$$

$$\dot{p} = \frac{4}{3} \sigma_T \frac{1}{8\pi} \frac{B^2 p^2}{(m_0 c)^2}$$

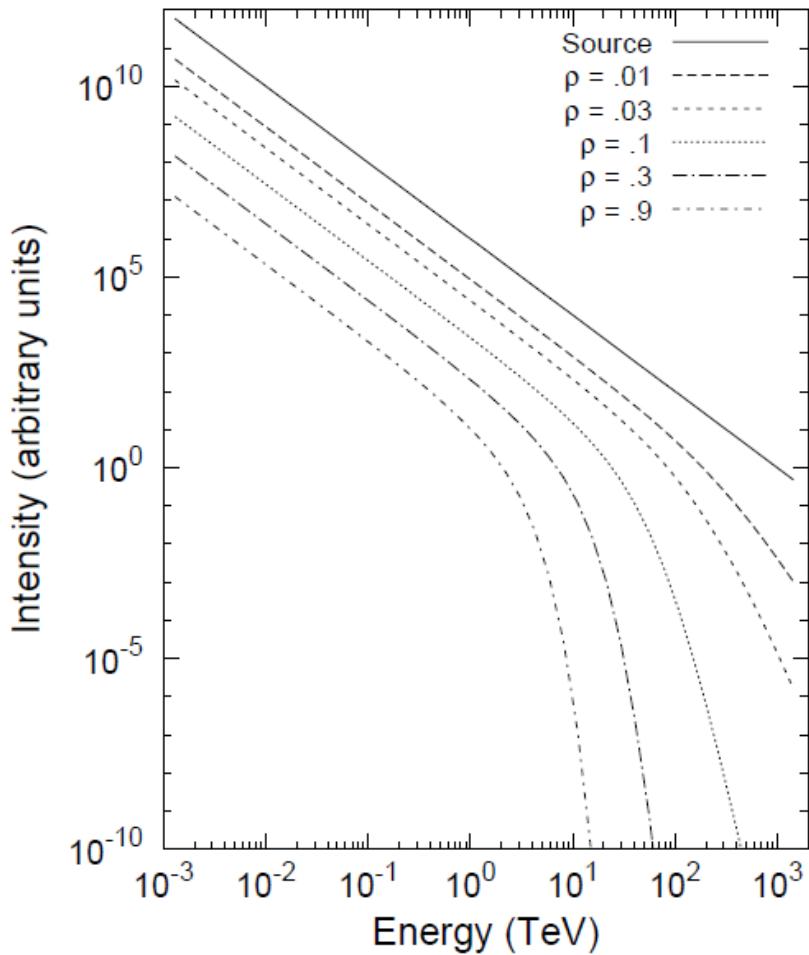
- Spherically symmetric:

$$\begin{aligned} \frac{\partial f}{\partial t} &= \\ &\kappa_r \frac{\partial^2 f}{\partial r^2} + \left[ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \kappa_r) - v_r \right] \frac{\partial f}{\partial r} + \frac{1}{3} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + 3k B^2 p \right] \frac{\partial f}{\partial \ln p} \\ &+ 4kB^2 p f + Q(r, p, t). \end{aligned}$$

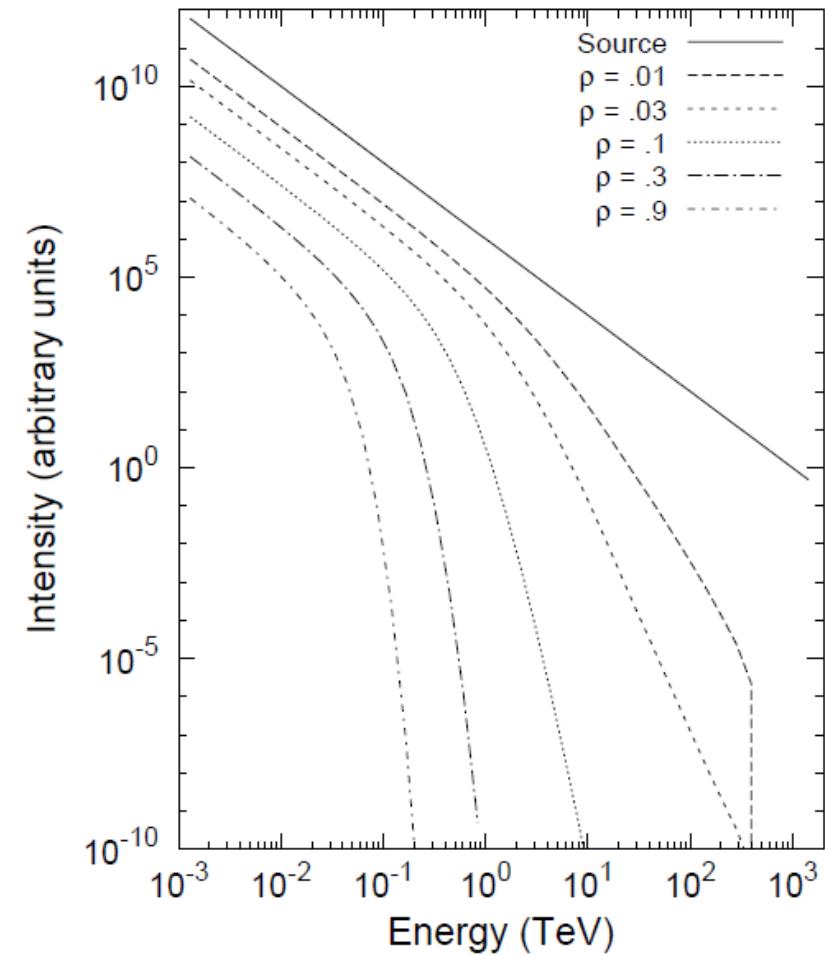
## *Time-independent: adiabatic losses*



## *Time-independent: synchrotron losses*

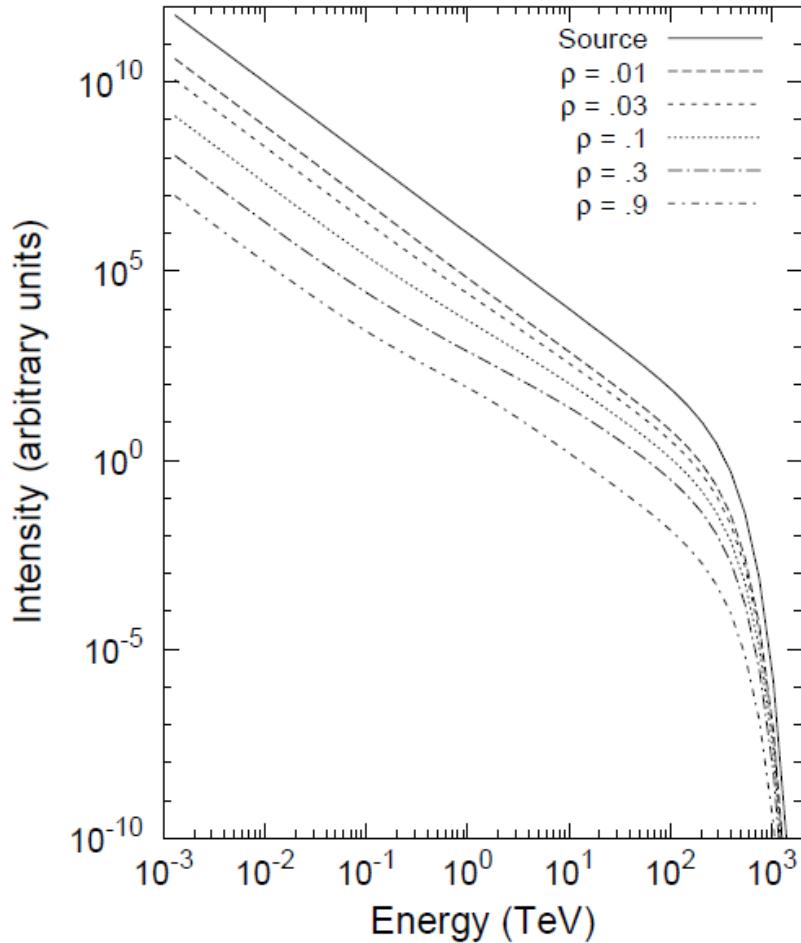


(a) Synchrotron losses for  $B = 100 \mu\text{G}$

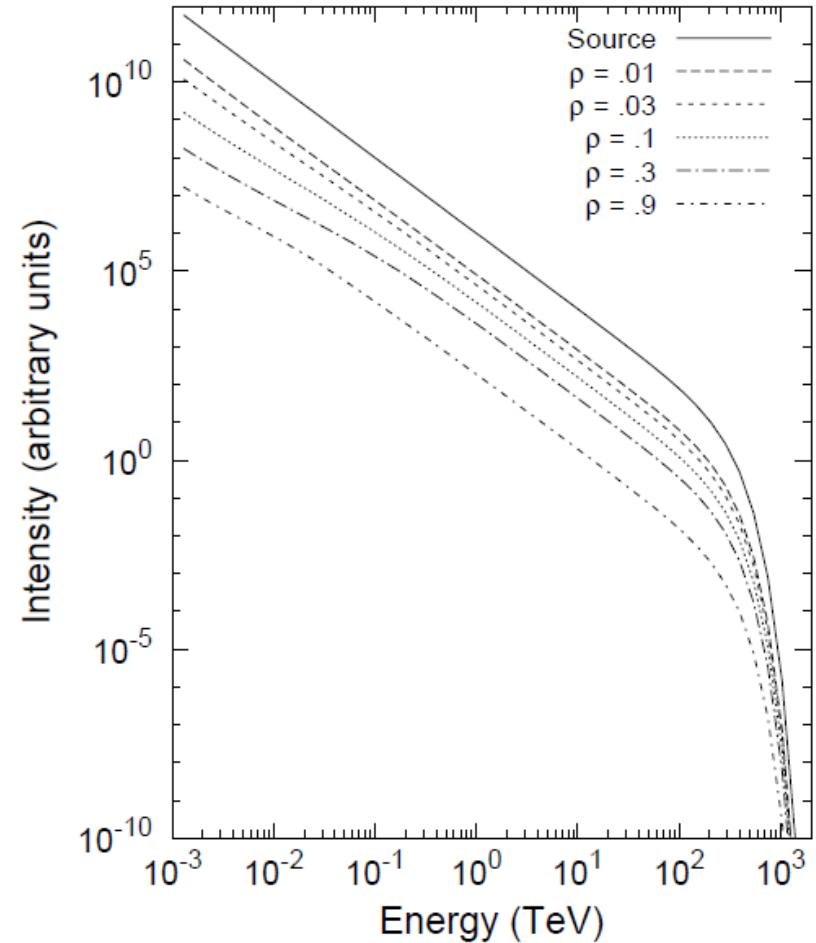


(b) Synchrotron losses for  $B = 1000 \mu\text{G}$

## *Time-independent: diffusion*



(a) Spectra calculated with the diffusion coefficient set to  $\kappa = 0.01\xi$ .



(b) Spectra calculated with the diffusion coefficient set to  $\kappa = 1\xi$ .

# Conclusions

- (M)HD model
  - results agree with observations
- Transport model
  - hyperbolic/parabolic nature of problem
  - diffusion can explain observed break in synchrotron spectrum